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BOND CASH FLOWS LITERACY INTERMEDIATE BOND MATH (PART 1)

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August 7, 2014

2:00 PM – 3:30 PM

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BOND CASH FLOWS LITERACY

INTERMEDIATE BOND MATH (PART 1)

PRESENTED BY LOUIS CHOI
PUBLIC RESOURCES ADVISORY GROUP
AN INDEPENDENT REGISTERED MUNICIPAL ADVISOR (IRMA)

Topics

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- Bonds and Loans
- How Municipal Bonds are Priced (or Valued)
- Understanding Cash Flow Schedules
- Debt Amortization
- Bonus: Using Microsoft Excel Functions

Bonds and Loans

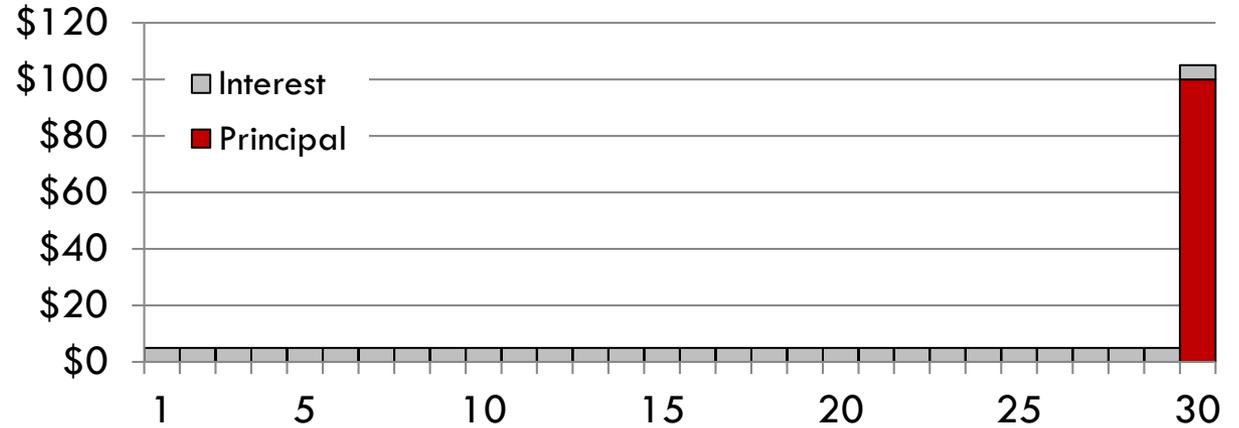
Bond cash flows literacy

intermediate bond math (Part 1)

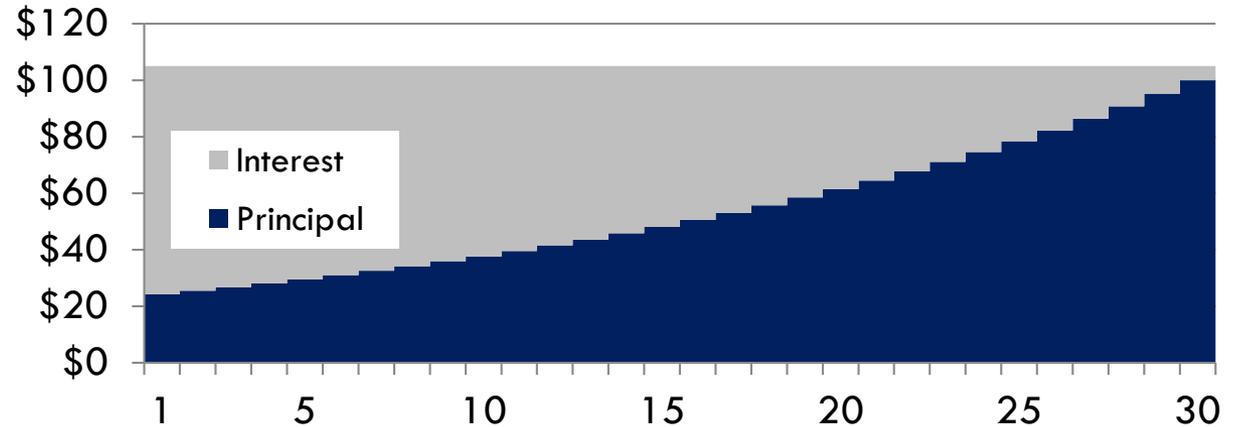
Bonds as Loans

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An investment...

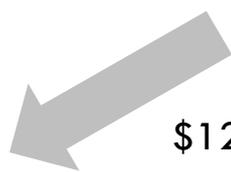
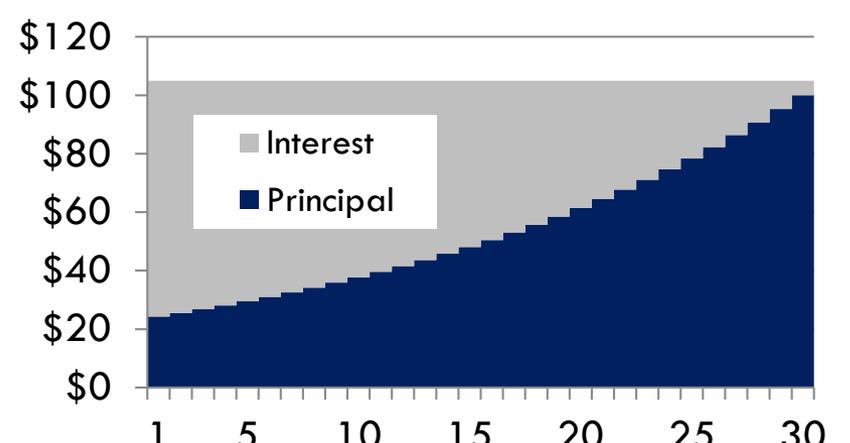
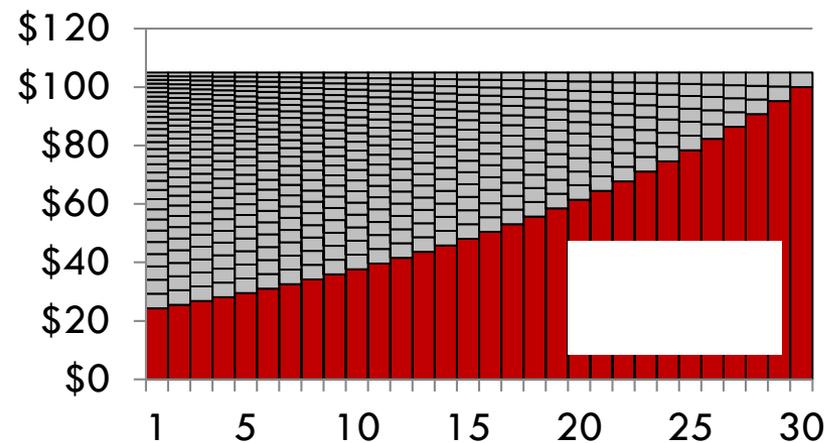
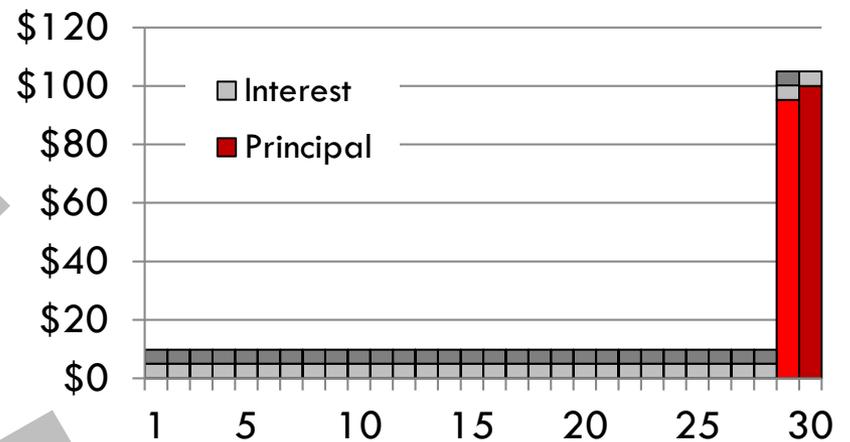
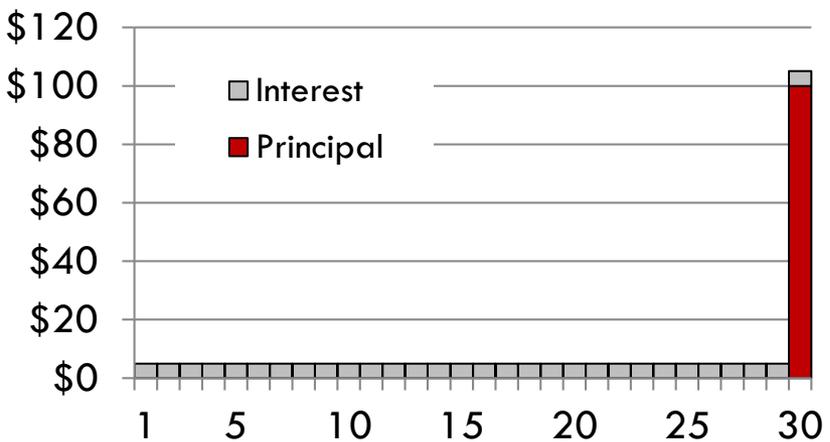


A loan...



In Aggregate, Bonds in an Issue Are Equivalent to a Loan

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A Bond Issue and a Loan Are Mathematically Similar, But Not Identical

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A Loan:

Date	Principal Balance	Principal	5.00% Interest	Debt Service
5/1/2014	50,000,000			
5/1/2015	50,000,000	9,050,000	2,500,000	11,550,000
5/1/2016	40,950,000	9,500,000	2,047,500	11,547,500
5/1/2017	31,450,000	9,975,000	1,572,500	11,547,500
5/1/2018	21,475,000	10,475,000	1,073,750	11,548,750
5/1/2019	11,000,000	11,000,000	550,000	11,550,000
Total		50,000,000	7,743,750	57,743,750

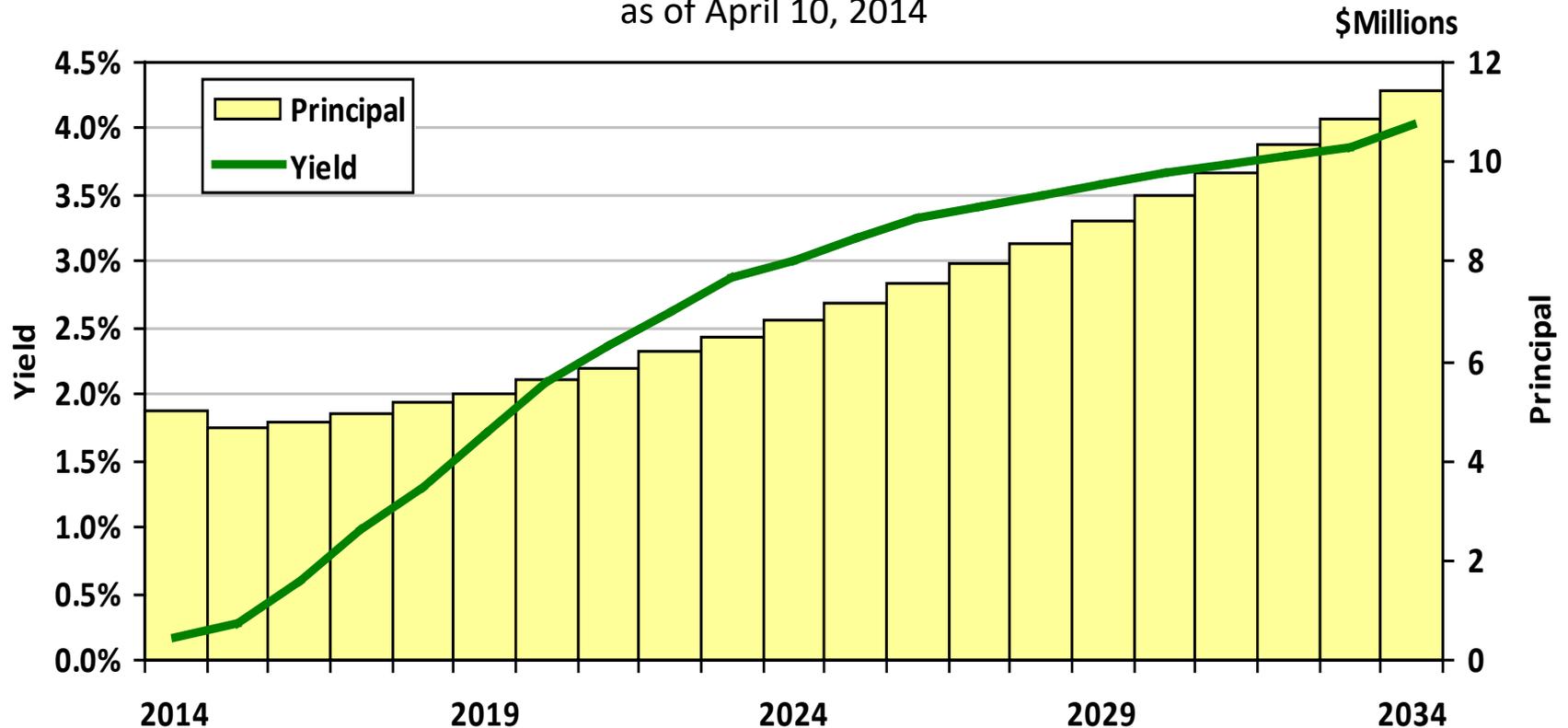
A Bond Issue:

Date	Principal Balance	Principal	Coupon	Interest on Principal Due					Total Interest	Debt Service
				2.00% 5/1/2015	3.00% 5/1/2016	4.00% 5/1/2017	4.50% 5/1/2018	5.00% 5/1/2019		
5/1/2014	50,000,000									
5/1/2015	50,000,000	9,050,000	2.00%	181,000	285,000	399,000	471,375	550,000	1,886,375	10,936,375
5/1/2016	40,950,000	9,500,000	3.00%		285,000	399,000	471,375	550,000	1,705,375	11,205,375
5/1/2017	31,450,000	9,975,000	4.00%			399,000	471,375	550,000	1,420,375	11,395,375
5/1/2018	21,475,000	10,475,000	4.50%				471,375	550,000	1,021,375	11,496,375
5/1/2019	11,000,000	11,000,000	5.00%					550,000	550,000	11,550,000
Total		50,000,000		181,000	570,000	1,197,000	1,885,500	2,750,000	6,583,500	56,583,500

Differences in Rates Across Maturities Generate a Yield Curve

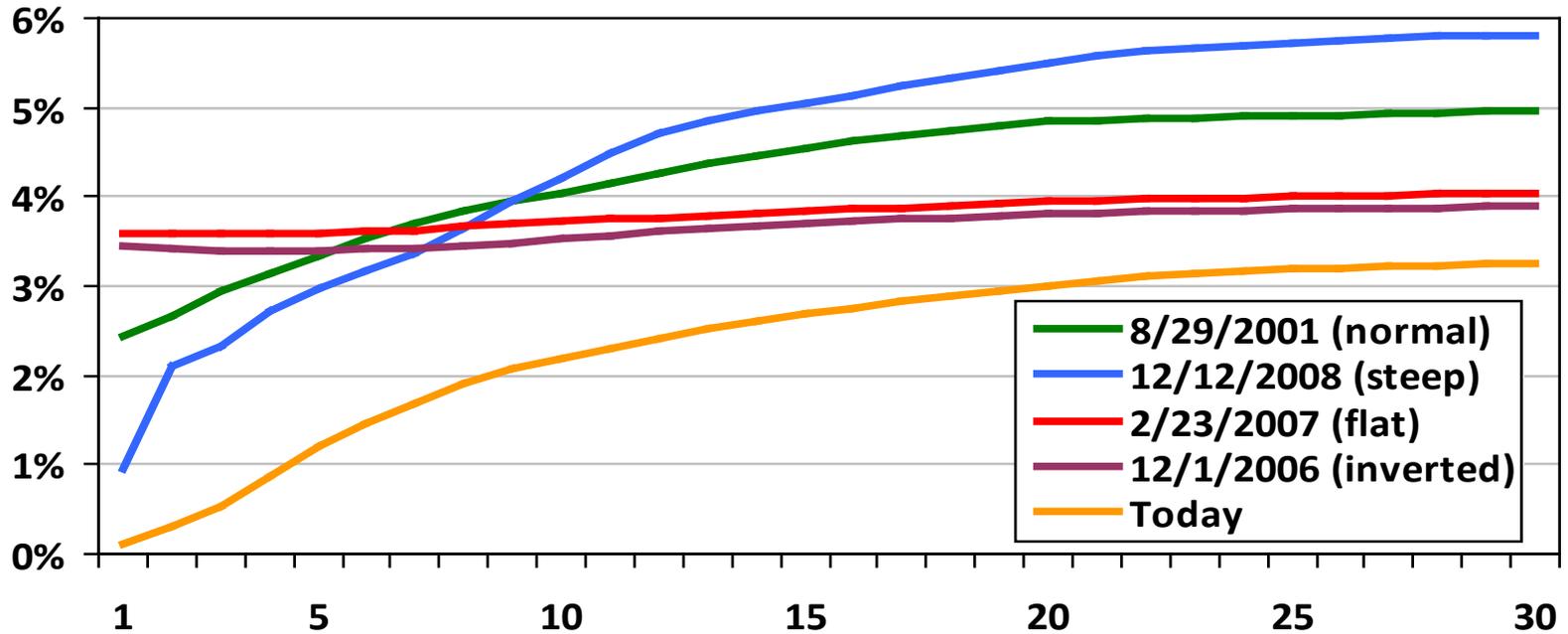
State Public Works Board of the State of California \$152,420,000 Lease Revenue Bonds (Department of Corrections and Rehabilitation), 2014 Series C

Principal Amounts and Initial Reoffering Yields
as of April 10, 2014



Selected Historical Yield Curves

AAA GO MMD



Percentile/Date	25th	50th	75th	8/29/01	12/12/08	2/23/07	12/1/06
1Y vs. 5Y MMD	0.52%	0.88%	1.30%	0.88%	2.01%	0.00%	-0.05%
1Y vs. 10Y MMD	0.95%	1.55%	2.35%	1.59%	3.26%	0.12%	0.08%
1Y vs. 30Y MMD	1.65%	2.50%	3.44%	2.50%	4.86%	0.44%	0.44%

Calculating Bond Prices

Bond cash flows literacy

intermediate bond math (Part 1)

Time-Value of Money (TVM)

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Calculates the value of future-day dollars in present-day dollars, and applicable to calculations for:

- Opportunity cost*
- Inflation*
- Investments*

Present Value Formula:

$$PV = \frac{FV}{\left(1 + \frac{i}{p}\right)^t}$$

- “PV” = Present Value
- “FV” = Future Cash Flows
- “ i ” = Interest Rate
- “ p ” = Compounding Periods Per Year
- “ t ” = Time or Periods

TVM Is the Basis for Calculating Bond Prices

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A stream of future cash flows, such as the periodic payment of interest and final payment of principal, follows the same approach as the sum of multiple terms

□ Present Value Formula for Multiple Future Cash Flows:

$$PV = \frac{CF_1}{\left(1 + \frac{i}{p}\right)^{t_1}} + \frac{CF_2}{\left(1 + \frac{i}{p}\right)^{t_2}} + \dots + \frac{CF_n}{\left(1 + \frac{i}{p}\right)^{t_n}}$$

- “PV” = Present Value, or Price
- “CF” = Future Cash Flows, which for bonds include:
 - ✓ Principal
 - ✓ Semi-Annual Interest
- “i” = Interest Rate, or Yield
- “p” = Compounding Periods Per Year
 - ✓ (Municipal Convention = 2)
- “t” = Time or Periods
 - ✓ (Municipal Convention = $\frac{30 \text{ Days}}{360} = \frac{1}{12}$)

Bond Pricing Formula

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□ Municipal Standard Price Formula:

$$P = \left[\frac{RV}{\left(1 + \frac{Y}{2}\right)^{\left(N - 1 + \frac{E-A}{E}\right)}} \right] + \left[\sum_{k=1}^N \frac{100 * \frac{R}{2}}{\left(1 + \frac{Y}{2}\right)^{\left(k - 1 + \frac{E-A}{E}\right)}} \right] - \left[100 * \frac{A}{B} * R \right]$$

**Principal
Component**

**Interest
Component**

**Accrued
Interest
Component**

- “A” = 30/360 days from dated date to settlement date
- “B” = Days in the year (usually 360)
- “E” = Days in semi-annual period (usually 180)
- “N” = Interest payments between settlement and redemption dates
- “P” = Dollar price (as a %)
- “R” = Annual coupon (as decimal)
- “RV” = Redemption value, including premiums, if any
- “Y” = Yield (as decimal)

*Expressed as a percentage
of the principal amount*

Prices Can Vary Greatly with Different Coupons and Maturities

10-year bond with a 3% coupon at yield of 3.15%

$$\left[\frac{100}{\left(1 + \frac{3.15\%}{2}\right)^{\left(20 - 1 + \frac{180-0}{180}\right)}} \right] + \left[\sum_{k=1}^{20} \frac{100 * \frac{3\%}{2}}{\left(1 + \frac{3.15\%}{2}\right)^{\left(k - 1 + \frac{180-0}{180}\right)}} \right] - \left[100 * \frac{0}{360} * 3\% \right] = 98.721$$

10-year bond with a 5% coupon at yield of 3.15%

$$\left[\frac{100}{\left(1 + \frac{3.15\%}{2}\right)^{\left(20 - 1 + \frac{180-0}{180}\right)}} \right] + \left[\sum_{k=1}^{20} \frac{100 * \frac{5\%}{2}}{\left(1 + \frac{3.15\%}{2}\right)^{\left(k - 1 + \frac{180-0}{180}\right)}} \right] - \left[100 * \frac{0}{360} * 5\% \right] = 115.764$$

10-year bond with a 5% coupon at yield of 5.165%

$$\left[\frac{100}{\left(1 + \frac{5.165\%}{2}\right)^{\left(20 - 1 + \frac{180-0}{180}\right)}} \right] + \left[\sum_{k=1}^{20} \frac{100 * \frac{5\%}{2}}{\left(1 + \frac{5.165\%}{2}\right)^{\left(k - 1 + \frac{180-0}{180}\right)}} \right] - \left[100 * \frac{0}{360} * 5\% \right] = 98.721$$

Terminology:

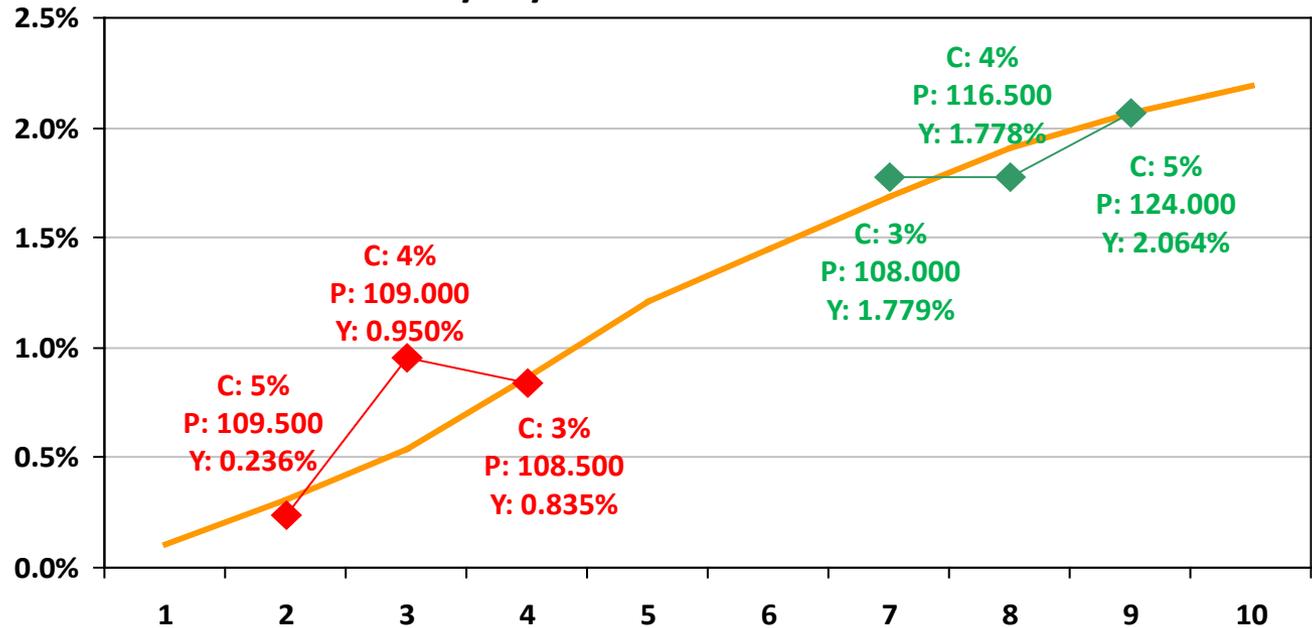
- Par: Price = 100
- Discount: Price < 100
- Premium: Price > 100

Bond Prices are Commonly Expressed in Yields for Ease of Comparison

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Yields help to inform consistency of pricing as terms vary

7/25/2014 AAA GO MMD



	Yield >>	0.11%	0.31%	0.54%	0.87%	1.21%	1.45%	1.69%	1.91%	2.07%	2.19%
Coupon	3.00%	102.887	105.359	107.310	108.355	108.659	108.876	108.614	108.050	107.600	107.239
	4.00%	103.886	107.351	110.282	112.278	113.496	114.602	115.189	115.437	115.773	116.176
	5.00%	104.885	109.343	113.254	116.201	118.334	120.329	121.765	122.823	123.946	125.113

Bond Pricing Conventions

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Using the price formula when coupon equals yield may result in a calculated price of 99.998 or 99.999

Guarantees investors that the stated yield would be achieved, regardless of whether or when the issuer exercises its option

Prices do not have to be calculated for every date; instead, only first dates when redemption prices change must be checked

- Bonds where coupon equals yield are priced at 100.000 (or par)
- Prices are truncated to third place after decimal
 - ▣ Ex.: price of 107.186243... becomes 107.186
 - ▣ Ex.: price of 98.53293... becomes 98.532
- Yields are rounded to the nearest third place after decimal
 - ▣ Ex.: yield of 5.16435...% becomes 5.164%
 - ▣ Ex.: yield of 3.18987...% becomes 3.190%
- For optionally callable premium bonds (i.e., coupon > yield), bonds are priced to that call date which results in the lowest price
 - ▣ Ex.: 11/1/2028 maturity, 4.2% coupon, 3.15% yield, callable on 11/1/2024 at 102, on 11/1/2025 at 101 and on 11/1/2026 at 100, and settled on 11/1/2014

Assumed Redemption Date	No. of interest periods (N)	Redemption value (RV)	Price (P)
11/1/2024	20	102	110.410
11/1/2025	22	101	110.406
11/1/2026	24	100	110.424
11/1/2028	28	100	111.813

Capital Appreciation Bonds (CAB)

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Also based on TVM formula

- ❑ Interest is compounded and paid at maturity
 - Growth in value of a CAB is expressed as an accreted value

$$AV_n = PR \times \left[1 + \frac{Y}{2} \right]^{(n \times 2)}$$

Note: Prices may be expressed as percentage of delivery date principal amount or final maturity amount, depending on how issuance principal is expressed

- “AV_n” = Accreted value at period n
 - “PR” = Initial price (generally par)
 - “Y” = Yield
- ❑ Generally, not subject to optional redemption
 - ❑ Sold in denominations such that the final accreted value of each denomination is \$5,000

Capital Appreciation Bonds (Cont'd)

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□ Solving for an accretion table...

$$AV_n = \frac{DN}{\left[1 + \frac{Y}{2}\right]^{((M-n) \times 2)}}$$

- “AV_n” = Accreted value at period n
- “DN” = Accreted value at maturity (effective denomination)
- “Y” = Yield
- “M” = Maturity

Example:

Delivery Date: 5/14/2014

Maturity: 5/1/2019

Yield: 3.50%

Effective Denomination: \$5,000

Date	Accreted Value
5/14/2014	\$4,208.91
11/1/2014	4,277.21
5/1/2015	4,352.06
11/1/2015	4,428.22
5/1/2016	4,505.71
11/1/2016	4,584.56
5/1/2017	4,664.79
11/1/2017	4,746.43
5/1/2018	4,829.49
11/1/2018	4,914.00
5/1/2019	5,000.00

Cash Flow Schedules

Bond cash flows literacy

intermediate bond math (Part 1)

Describing a Bond Issue with Numbers

Goal: to understand how the numbers that describe individual bonds and a bond issue work

MATURITIES, PRINCIPAL AMOUNTS, INTEREST RATES, YIELDS AND CUSIPs

\$28,790,000 2012B Serial Bonds

Maturity Date (June 1)	Principal Amount	Interest Rate	Yield	CUSIP ¹	Maturity Date (June 1)	Principal Amount	Interest Rate	Yield
2015	\$1,075,000	4.000%	0.830%	1306853D0	2024	\$1,575,000	5.000%	3.010% ^C
2016	1,120,000	4.000	1.090	1306853E8	2025	1,655,000	5.000	3.200C
2017	1,165,000	4.000	1.350	1306853F5	2026	1,735,000	5.000	3.320C
2018	1,210,000	4.000	1.620	1306853G3	2027	1,825,000	5.000	3.410C
2019	1,255,000	4.000	1.890	1306853H1	2028	1,915,000	5.000	3.490C
2020	1,310,000	4.000	2.170	1306853J7	2029	2,010,000	5.000	3.570C
2021	1,360,000	5.000	2.400	1306853K4	2030	2,110,000	5.000	3.640C
2022	1,430,000	5.000	2.570	1306853L2	2031	2,215,000	5.000	3.700C
2023	1,500,000	5.000	2.770 ^C	1306853M0	2032	2,325,000	5.000	3.740C

\$13,260,000 4.125% 2012 Series B Term Bonds due June 1, 2037, Yield: 4.240%, CUSIP¹: 1306853X6

Estimated Sources and Uses of Funds

The proceeds to be received from the sale of the 2012B Bonds are expected to be applied as set forth below:

Estimated Sources

Principal Amount of 2012B Bonds	\$42,050,000.00
Plus Net Original Issue Premium	<u>3,789,642.70</u>
Total Estimated Sources	<u>\$45,839,642.70</u>

Estimated Uses

Project Account	\$39,595,000.00
Capitalized Interest ⁽¹⁾	5,931,575.09
Costs of Issuance ⁽²⁾	131,795.97
Underwriters' Discount	<u>181,271.64</u>
Total Estimated Uses	<u>\$45,839,642.70</u>

⁽¹⁾ Funded to pay interest on the 2012B Bonds to the date which is three months after the expected construction completion date for the 2012B Project.

⁽²⁾ Includes the State Treasurer's fees for serving as trustee, legal and rating agencies' fees, and other costs of issuance, including Board administration fees.

Annual Fiscal Year Debt Service Requirements

Set forth below are the principal, interest and total debt service requirements for the 2012B Bonds, assuming no redemptions other than scheduled mandatory sinking account redemptions:

Payment Date	2012B Bonds Principal	2012B Bonds Interest	Total 2012B Bonds Debt Service	Annual Fiscal Year Debt Service
12/1/2012		\$1,143,775.85	\$1,143,775.85	
6/1/2013		957,549.24	957,549.24	\$2,101,325.09
12/1/2013		957,575.76	957,575.76	
6/1/2014		957,549.24	957,549.24	1,915,125.00
12/1/2014		957,575.76	957,575.76	
6/1/2015	\$1,075,000	957,549.24	2,032,549.24	2,990,125.00
12/1/2015		936,075.76	936,075.76	
6/1/2016	1,120,000	936,049.24	2,056,049.24	2,992,125.00
12/1/2016		913,675.76	913,675.76	
6/1/2017	1,165,000	913,649.24	2,078,649.24	2,992,325.00
12/1/2017		890,375.76	890,375.76	
6/1/2018	1,210,000	890,349.24	2,100,349.24	2,990,725.00
12/1/2018		866,175.76	866,175.76	
6/1/2019	1,255,000	866,149.24	2,121,149.24	2,987,325.00
12/1/2019		841,075.76	841,075.76	
6/1/2020	1,310,000	841,049.24	2,151,049.24	2,992,125.00
12/1/2020		814,875.76	814,875.76	
6/1/2021	1,360,000	814,849.24	2,174,849.24	2,989,725.00
12/1/2021		780,875.76	780,875.76	
6/1/2022	1,430,000	780,849.24	2,210,849.24	2,991,725.00
12/1/2022		745,125.76	745,125.76	
6/1/2023	1,500,000	745,099.24	2,245,099.24	2,990,225.00
12/1/2023		707,625.76	707,625.76	
6/1/2024	1,575,000	707,599.24	2,282,599.24	2,990,225.00
12/1/2024		668,250.76	668,250.76	
6/1/2025	1,655,000	668,224.24	2,323,224.24	2,991,475.00
12/1/2025		626,875.76	626,875.76	
6/1/2026	1,735,000	626,849.24	2,361,849.24	2,988,725.00
12/1/2026		583,500.76	583,500.76	
6/1/2027	1,825,000	583,474.24	2,408,474.24	2,991,975.00
12/1/2027		537,875.76	537,875.76	
6/1/2028	1,915,000	537,849.24	2,452,849.24	2,990,725.00
12/1/2028		490,000.76	490,000.76	
6/1/2029	2,010,000	489,974.24	2,499,974.24	2,989,975.00
12/1/2029		439,750.76	439,750.76	
6/1/2030	2,110,000	439,724.24	2,549,724.24	2,989,475.00
12/1/2030		387,000.76	387,000.76	
6/1/2031	2,215,000	386,974.24	2,601,974.24	2,988,975.00
12/1/2031		331,625.76	331,625.76	
6/1/2032	2,325,000	331,599.24	2,656,599.24	2,988,225.00
12/1/2032		273,500.76	273,500.76	
6/1/2033	2,440,000	273,474.24	2,713,474.24	2,986,975.00
12/1/2033		223,173.32	223,173.32	
6/1/2034	2,545,000	223,151.68	2,768,151.68	2,991,325.00
12/1/2034		170,680.15	170,680.15	
6/1/2035	2,650,000	170,663.60	2,820,663.60	2,991,343.75
12/1/2035		116,021.25	116,021.25	
6/1/2036	2,755,000	116,010.00	2,871,010.00	2,987,031.25
12/1/2036		59,196.62	59,196.62	
6/1/2037	2,870,000	59,190.88	2,929,190.88	2,988,387.50
	\$42,050,000	\$30,737,712.59	\$72,787,712.59	\$72,787,712.59

Start with a Basic Loan...

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			5.00%	
	Principal		Interest on	Debt
Date	Balance	Principal	Balance	Service
5/1/2014	50,000,000			
5/1/2015	50,000,000	9,048,740	2,500,000	11,548,740
5/1/2016	40,951,260	9,501,177	2,047,563	11,548,740
5/1/2017	31,450,083	9,976,236	1,572,504	11,548,740
5/1/2018	21,473,847	10,475,048	1,073,692	11,548,740
5/1/2019	10,998,800	10,998,800	549,940	11,548,740
Total		50,000,000	7,743,700	57,743,700

Assumptions –

- \$50,000,000 borrowed
- Repaid in 5 years
- Interest rate of 5.00%

...Round by Denominations...

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	Principal		5.00%	
	Balance	Principal	Interest on	Debt
Date	Balance	Principal	Balance	Service
5/1/2014	50,000,000			
5/1/2015	50,000,000	9,050,000	2,500,000	11,550,000
5/1/2016	40,950,000	9,500,000	2,047,500	11,547,500
5/1/2017	31,450,000	9,975,000	1,572,500	11,547,500
5/1/2018	21,475,000	10,475,000	1,073,750	11,548,750
5/1/2019	11,000,000	11,000,000	550,000	11,550,000
Total		50,000,000	7,743,750	57,743,750

Municipal bonds are generally sold (and therefore repaid) in denominations of \$5,000

...Reflect Different Interest Rates (Coupons) for Each Maturity...

Date	Principal Balance	Principal	Coupon	2.00%	3.00%	4.00%	5.00%	5.00%	Total Interest	Debt Service
				Interest on Principal Due						
				5/1/2015	5/1/2016	5/1/2017	5/1/2018	5/1/2019		
5/1/2014	50,000,000									
5/1/2015	50,000,000	9,050,000	2.00%	181,000	285,000	399,000	523,750	550,000	1,938,750	10,988,750
5/1/2016	40,950,000	9,500,000	3.00%		285,000	399,000	523,750	550,000	1,757,750	11,257,750
5/1/2017	31,450,000	9,975,000	4.00%			399,000	523,750	550,000	1,472,750	11,447,750
5/1/2018	21,475,000	10,475,000	5.00%				523,750	550,000	1,073,750	11,548,750
5/1/2019	11,000,000	11,000,000	5.00%					550,000	550,000	11,550,000
Total		50,000,000		181,000	570,000	1,197,000	2,095,000	2,750,000	6,793,000	56,793,000

Or in the more familiar form below:

Date	Principal Balance	Principal	Coupon	Total Interest	Debt Service
5/1/2014	50,000,000				
5/1/2015	50,000,000	9,050,000	2.00%	1,938,750	10,988,750
5/1/2016	40,950,000	9,500,000	3.00%	1,757,750	11,257,750
5/1/2017	31,450,000	9,975,000	4.00%	1,472,750	11,447,750
5/1/2018	21,475,000	10,475,000	5.00%	1,073,750	11,548,750
5/1/2019	11,000,000	11,000,000	5.00%	550,000	11,550,000
Total		50,000,000		6,793,000	56,793,000

...Adjust Principal of Each Maturity to Achieve Debt Service Pattern...

24

Date	Principal Balance	Principal	Coupon	2.00%	3.00%	4.00%	5.00%	5.00%	Total Interest	Debt Service
				Interest on Principal Due						
				5/1/2015	5/1/2016	5/1/2017	5/1/2018	5/1/2019		
5/1/2014	50,000,000									
5/1/2015	50,000,000	9,415,000	2.00%	188,300	288,150	395,600	514,500	540,000	1,926,550	11,341,550
5/1/2016	40,585,000	9,605,000	3.00%		288,150	395,600	514,500	540,000	1,738,250	11,343,250
5/1/2017	30,980,000	9,890,000	4.00%			395,600	514,500	540,000	1,450,100	11,340,100
5/1/2018	21,090,000	10,290,000	5.00%				514,500	540,000	1,054,500	11,344,500
5/1/2019	10,800,000	10,800,000	5.00%					540,000	540,000	11,340,000
Total		50,000,000		188,300	576,300	1,186,800	2,058,000	2,700,000	6,709,400	56,709,400

Once again, or in the more familiar form below:

Date	Principal Balance	Principal	Coupon	Total Interest	Debt Service
5/1/2014	50,000,000				
5/1/2015	50,000,000	9,415,000	2.00%	1,926,550	11,341,550
5/1/2016	40,585,000	9,605,000	3.00%	1,738,250	11,343,250
5/1/2017	30,980,000	9,890,000	4.00%	1,450,100	11,340,100
5/1/2018	21,090,000	10,290,000	5.00%	1,054,500	11,344,500
5/1/2019	10,800,000	10,800,000	5.00%	540,000	11,340,000
Total		50,000,000		6,709,400	56,709,400

...Introduce Prices, Yields and Proceeds...

25

Date	Principal Balance	Principal	Coupon	Total Interest	Debt Service	Yield	Price	Proceeds
5/1/2014	50,000,000							
5/1/2015	50,000,000	9,415,000	2.00%	1,926,550	11,341,550	1.00%	100.992	9,508,397
5/1/2016	40,585,000	9,605,000	3.00%	1,738,250	11,343,250	1.75%	102.446	9,839,938
5/1/2017	30,980,000	9,890,000	4.00%	1,450,100	11,340,100	2.25%	105.049	10,389,346
5/1/2018	21,090,000	10,290,000	5.00%	1,054,500	11,344,500	2.75%	108.467	11,161,254
5/1/2019	10,800,000	10,800,000	5.00%	540,000	11,340,000	3.10%	108.737	11,743,596
Total		50,000,000		6,709,400	56,709,400			52,642,532

Each maturity generates proceeds equal to the product of its price and its principal.

Note: Prices are calculated following all of the rules discussed above.

...Calculate Purchase Price...

Date	Principal Balance	Principal	Coupon	Total Interest	Debt Service	Yield	Price	Proceeds	Takedown (\$/\$1,000)	Takedown (\$)
5/1/2014	50,000,000									
5/1/2015	50,000,000	9,415,000	2.00%	1,926,550	11,341,550	1.00%	100.992	9,508,397	1.00	9,415
5/1/2016	40,585,000	9,605,000	3.00%	1,738,250	11,343,250	1.75%	102.446	9,839,938	2.50	24,013
5/1/2017	30,980,000	9,890,000	4.00%	1,450,100	11,340,100	2.25%	105.049	10,389,346	2.50	24,725
5/1/2018	21,090,000	10,290,000	5.00%	1,054,500	11,344,500	2.75%	108.467	11,161,254	3.75	38,588
5/1/2019	10,800,000	10,800,000	5.00%	540,000	11,340,000	3.10%	108.737	11,743,596	3.75	40,500
Total		50,000,000		6,709,400	56,709,400			52,642,532		137,240

The purchase price paid to the issuer is net of both compensation and expenses withheld by the underwriter

Principal	50,000,000
Net Original Issue Premium/(Discount)	2,642,532
Production	52,642,532
Underwriter's Discount	-162,052
Purchase Price	<u>52,480,479</u>
Underwriter's Discount	
Takedown	137,240
Underwriter's Counsel	15,000
CDIAC	3,000
CUSIP	600
Day Loan	1,462
Dalcomp	3,750
Dalnet	500
DTC	500
Total Uses of Funds	<u>162,052</u>

...Add in Sources and Uses Components...

27

Date	Principal Balance	Principal	Coupon	Total Interest	Debt Service	Yield	Price	Proceeds	Takedown (\$/\$1,000)	Takedown (\$)
5/1/2014	50,000,000									
5/1/2015	50,000,000	9,415,000	2.00%	1,926,550	11,341,550	1.00%	100.992	9,508,397	1.00	9,415
5/1/2016	40,585,000	9,605,000	3.00%	1,738,250	11,343,250	1.75%	102.446	9,839,938	2.50	24,013
5/1/2017	30,980,000	9,890,000	4.00%	1,450,100	11,340,100	2.25%	105.049	10,389,346	2.50	24,725
5/1/2018	21,090,000	10,290,000	5.00%	1,054,500	11,344,500	2.75%	108.467	11,161,254	3.75	38,588
5/1/2019	10,800,000	10,800,000	5.00%	540,000	11,340,000	3.10%	108.737	11,743,596	3.75	40,500
Total		50,000,000		6,709,400	56,709,400			52,642,532		137,240

Sources of Funds		Principal	Net Original Issue Premium/(Discount)	Production	Underwriter's Discount	Purchase Price					
Principal	50,000,000	50,000,000	2,642,532	52,642,532	-162,052	52,480,479					
Net OIP / (OID)	2,642,532										
Funds on Hand	1,000,000										
Total Sources of Funds	53,642,532										
Uses of Funds		Underwriter's Discount	Takedown	Underwriter's Counsel	CDIAC	CUSIP	Day Loan	Dalcomp	Dalnet	DTC	Total Uses of Funds
Project Deposit	50,000,000		137,240	15,000	3,000	600	1,462	3,750	500	500	162,052
Reserve Fund	5,264,253										
Costs of Issuance	240,000										
Underwriter's Discount	162,052										
Contingency	-2,023,774										
Total Uses of Funds	53,642,532										

Project deposit represents target amount to be borrowed

Notes: Reserve fund is generally equal to the least of: 10% of proceeds, maximum annual debt service (MADS) and 125% of average annual debt service. Contingency is a positive number that is less than the minimum denomination, adjusted by the prices of the bonds

...and Readjust Principal of Each Maturity to Target Proceeds

Date	Principal Balance	Principal	Coupon	Total Interest	Debt Service	Yield	Price	Proceeds	Takedown (\$/\$1,000)	Takedown (\$)
5/1/2014	50,000,000									
5/1/2015	50,000,000	9,820,000	2.00%	2,009,200	11,829,200	1.00%	100.992	9,917,414	1.00	9,820
5/1/2016	40,180,000	10,015,000	3.00%	1,812,800	11,827,800	1.75%	102.446	10,259,967	2.50	25,038
5/1/2017	30,165,000	10,315,000	4.00%	1,512,350	11,827,350	2.25%	105.049	10,835,804	2.50	25,788
5/1/2018	19,850,000	10,730,000	5.00%	1,099,750	11,829,750	2.75%	108.467	11,638,509	3.75	40,238
5/1/2019	9,120,000	11,265,000	5.00%	563,250	11,828,250	3.10%	108.737	12,249,223	3.75	42,244
Total		52,145,000		6,997,350	59,142,350			54,900,918		143,126

Sources of Funds		Principal	Net Original Issue Premium/(Discount)
Principal	52,145,000	52,145,000	2,755,918
Net OIP / (OID)	2,755,918		
Funds on Hand	1,000,000		
Total Sources of Funds	55,900,918		
Uses of Funds		Underwriter's Discount	Takedown
Project Deposit	50,000,000		143,126
Reserve Fund	5,490,092		
Costs of Issuance	240,000		
Underwriter's Discount	168,162		
Contingency	2,664		
Total Uses of Funds	55,900,918		
		Purchase Price	54,732,756
		Underwriter's Discount	
		Takedown	143,126
		Underwriter's Counsel	15,000
		CDIAC	3,000
		CUSIP	600
		Day Loan	1,525
		Dalcomp	3,911
		Dalnet	500
		DTC	500
		Total Uses of Funds	168,162

Note: Contingency should be greater than zero, but less than one denomination of the issued bond, after accounting for the prices of the bonds.

How to Calculate the “Yield” of a Bond Issue

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Date	Principal		Coupon	Total Interest	Debt Service	Present Value of Debt Service at		
	Balance	Principal				2.55204%	2.65880%	2.81677%
5/1/2014	50,000,000							
5/1/2015	50,000,000	9,820,000	2.00%	2,009,200	11,829,200	11,545,493	11,533,854	11,516,666
5/1/2016	40,180,000	10,015,000	3.00%	1,812,800	11,827,800	11,253,871	11,230,634	11,196,363
5/1/2017	30,165,000	10,315,000	4.00%	1,512,350	11,827,350	10,969,879	10,935,624	10,885,168
5/1/2018	19,850,000	10,730,000	5.00%	1,099,750	11,829,750	10,694,982	10,650,256	10,584,464
5/1/2019	9,120,000	11,265,000	5.00%	563,250	11,828,250	10,422,842	10,368,195	10,287,917
Total		52,145,000		6,997,350	59,142,350	54,887,069	54,718,564	54,470,579

Find the rate as the internal rate of return (IRR) of debt service to the target value

Sources of Funds				True		
				Arbitrage Yield	Interest Cost	All-in True Interest Cost
Principal		52,145,000				
Net OIP / (OID)		2,755,918				
Funds on Hand		1,000,000				
Total Sources of Funds		55,900,918				
			Proceeds	54,900,918	54,900,918	54,900,918
			Costs of Issuance			-240,000
			Underwriter's Discount		-168,162	-168,162
			Arbitrage Adjustments			
			Target Value	54,900,918	54,732,756	54,492,756
Uses of Funds						
Project Deposit		50,000,000				
Reserve Fund		5,490,092				
Costs of Issuance		240,000				
Underwriter's Discount		168,162				
Contingency		2,664				
Total Uses of Funds		55,900,918				

Note: Debt service may be required to be adjusted for bonds subject to redemption, when calculating the arbitrage yield

Arbitrage yield, true interest cost (TIC) and all-in TIC each represent a way to express the cost of capital for a bond issue

How to Calculate an “Average”

Date	Principal Balance	Principal	Coupon	Principal x Coupon	Total Interest	Debt Service	Years to Maturity	Principal x Years to Maturity
5/1/2014	50,000,000							
5/1/2015	50,000,000	9,820,000	2.00%	196,400	2,009,200	11,829,200	1.00000	9,820,000
5/1/2016	40,180,000	10,015,000	3.00%	300,450	1,812,800	11,827,800	2.00000	20,030,000
5/1/2017	30,165,000	10,315,000	4.00%	412,600	1,512,350	11,827,350	3.00000	30,945,000
5/1/2018	19,850,000	10,730,000	5.00%	536,500	1,099,750	11,829,750	4.00000	42,920,000
5/1/2019	9,120,000	11,265,000	5.00%	563,250	563,250	11,828,250	5.00000	56,325,000
Total		52,145,000		2,009,200	6,997,350	59,142,350		160,040,000

Average Coupon	2,009,200
52,145,000	
3.85%	
Weighted Average Maturity	160,040,000
52,145,000	
3.06913	

In general, averages are calculated as weighted averages by principal

Debt Amortization

Bond cash flows literacy

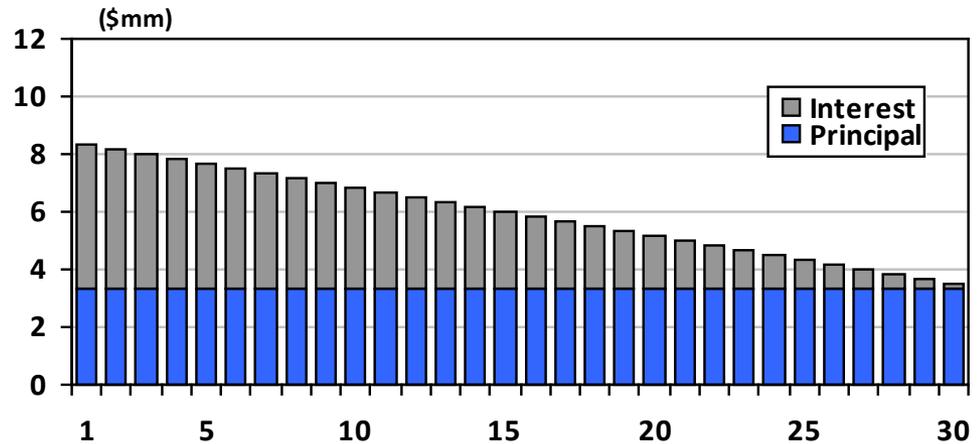
intermediate bond math (Part 1)

Common Amortization Structures

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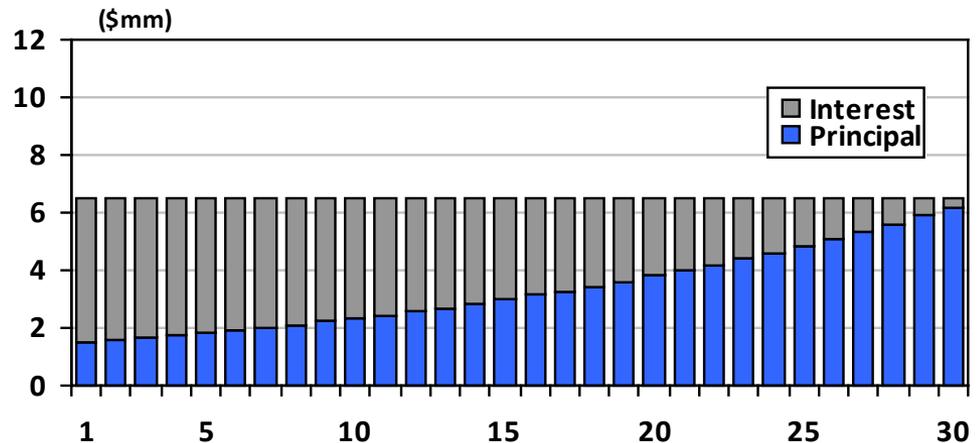
Level Principal:

- *Ease of calculation*
- *Common for bank product term-out provisions and GOs*
- *Interest/principal ratio: 0.77
(based on 5% rate)*



Level Debt Service:

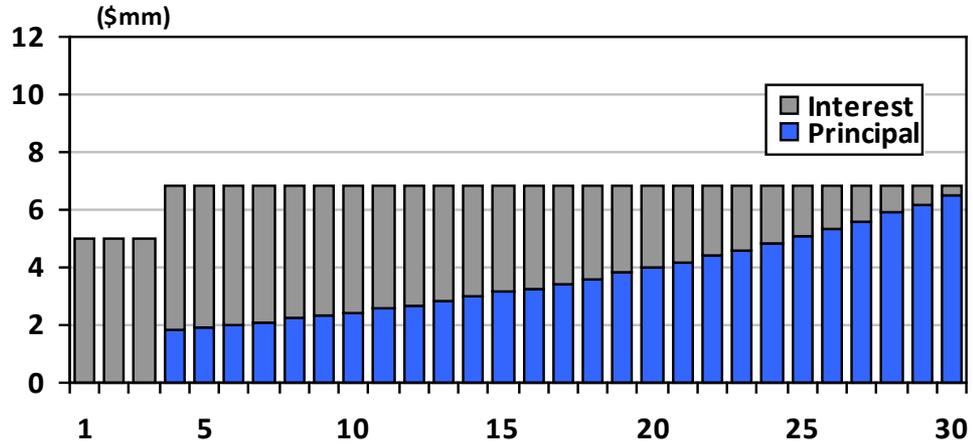
- *Even distribution of cost*
- *Simplify long-term budget preparation*
- *Interest/principal ratio: 0.95
(based on 5% rate)*



Common Amortization Structures

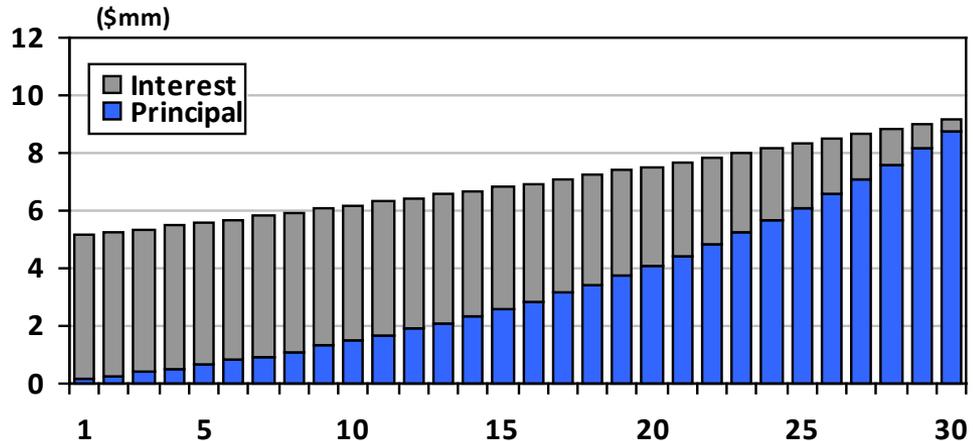
Deferred Principal:

- Revenues or operational cost savings become available at the later date (e.g. upon project completion)
- Interest/principal ratio: 0.99 (based on 5% rate)



Ascending Debt Service:

- Growing revenues
- Cost-recovery mechanism is subject to inflation
- Interest/principal ratio: 1.10 (based on 5% rate and 2% annual growth)

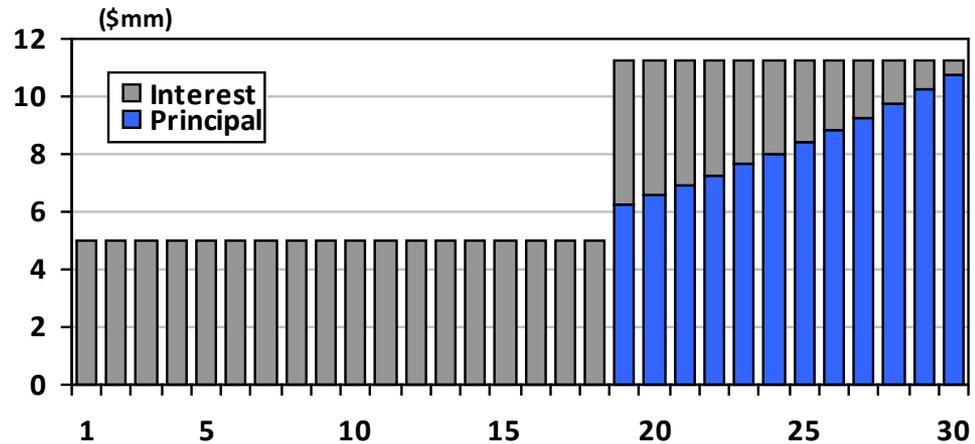


Common Amortization Structures

34

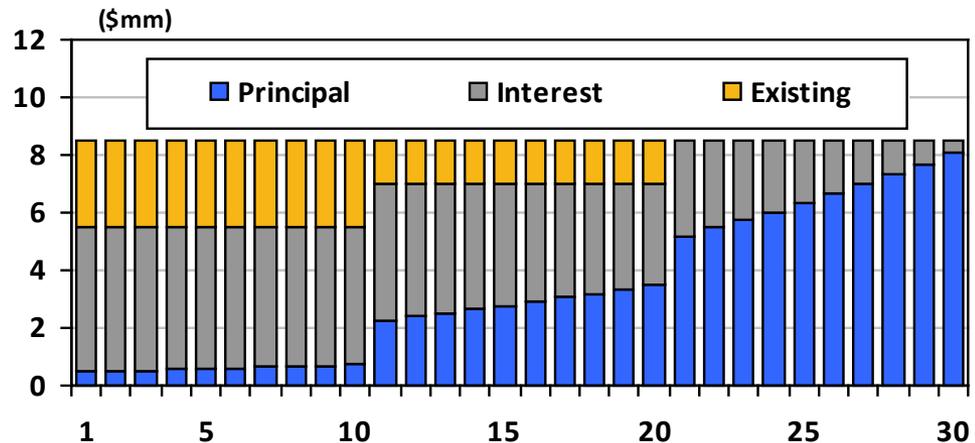
Backloaded Principal:

- Type of bond has the lowest expected cost of funds (e.g., floating rate or tax credit bonds)
- Interest/principal ratio: 1.25 (based on 5% rate, for THIS example)



Wrapped Debt Service:

- Profile of aggregate debt service is level
- Extends useful life of existing debt-funded asset
- Interest/principal ratio: 1.09 (based on 5% rate, for THIS example)



Solving for Amortization Structure

35

Debt service is equal to the sum of:

- *Principal;*
- *Interest on principal due; and*
- *Interest on principal still outstanding*

☐ Debt Service for a given year:

$$DS_n = (1 + C_n) \times P_n + \sum_{i=n+1} P_i \times C_i$$

- “DS_n” = Debt service for year n
- “P_n” = Principal amount for maturity n
- “C_n” = Coupon for maturity n

☐ Solving algebraically for principal results in the following:

$$P_n = \frac{DS_n - \sum_{i=n+1} P_i \times C_i}{1 + C_n}$$

☐ Given target debt service numbers, each principal amount can be solved

Solving for Amortization Structure

36

No unknowns!

□ Begin from the last maturity...

Example:

Target debt service: \$5,000,000

Coupon for 2024 (last maturity): 5.00%

$$P_{2018} = \frac{\$5,000,000 - \$0}{1 + 5.00\%}$$

$$P_{2018} = \$4,761,904.80 \quad \text{or round down to}$$

$$P_{2018} = \$4,760,000.00$$

Rounding
based on
denomination

Year	Target Debt Service	Principal	Coupon	Interest	Debt Service	Difference between Target and Actual D/S
2015	\$5,000,000	????	3.00%	238,000	238,000	4,762,000
2016	\$5,000,000	????	3.00%	238,000	238,000	4,762,000
2017	\$5,000,000	????	3.50%	238,000	238,000	4,762,000
2018	\$5,000,000	????	5.00%	238,000	238,000	4,762,000
2019	\$5,000,000	????	5.00%	238,000	238,000	4,762,000
2020	\$5,000,000	????	5.00%	238,000	238,000	4,762,000
2021	\$5,000,000	????	5.00%	238,000	238,000	4,762,000
2022	\$5,000,000	????	4.75%	238,000	238,000	4,762,000
2023	\$5,000,000	????	4.75%	238,000	238,000	4,762,000
2024	\$5,000,000	4,760,000	5.00%	238,000	4,998,000	2,000
Total	\$50,000,000	4,760,000		2,380,000	7,140,000	42,860,000

Solving for Amortization Structure (Cont'd)

37

...only one unknown...

...Continue with next to last maturity...

Example:

Target debt service : \$5,000,000

Coupon for 2023 (next to last maturity): 4.75%

Principal for 2024 (last maturity): \$4,760,000

Coupon for 2024 (last maturity): 5.00%

$$P_{2017} = \frac{\$5,000,000 - \$4,760,000 \times 5.00\%}{1 + 4.75\%}$$

$$P_{2017} = \$4,546,062.05 \quad \text{or round down to}$$

$$P_{2017} = \$4,545,000.00$$

*...which was just solved
in the last step!*

Year	Target Debt Service	Principal	Coupon	Interest	Debt Service	Difference between Target and Actual D/S
2015	\$5,000,000	????	3.00%	\$453,888	\$453,888	\$4,546,113
2016	\$5,000,000	????	3.00%	\$453,888	\$453,888	\$4,546,113
2017	\$5,000,000	????	3.50%	\$453,888	\$453,888	\$4,546,113
2018	\$5,000,000	????	5.00%	\$453,888	\$453,888	\$4,546,113
2019	\$5,000,000	????	5.00%	\$453,888	\$453,888	\$4,546,113
2020	\$5,000,000	????	5.00%	\$453,888	\$453,888	\$4,546,113
2021	\$5,000,000	????	5.00%	\$453,888	\$453,888	\$4,546,113
2022	\$5,000,000	????	4.75%	\$453,888	\$453,888	\$4,546,113
2023	\$5,000,000	\$4,545,000	4.75%	\$453,888	\$4,998,888	\$1,113
2024	\$5,000,000	\$4,760,000	5.00%	\$238,000	\$4,998,000	\$2,000
Total	\$50,000,000	\$9,305,000		\$4,322,988	\$13,627,988	\$36,372,013

Solving for Amortization Structure (Cont'd)

38

*Remaining unknowns
will be solved just in
time as well*

□ ...And so forth

Year	Target Debt			Interest	Debt Service	Difference between Target and Actual D/S
	Service	Principal	Coupon			
2015	\$5,000,000	\$3,250,000	3.00%	\$1,747,400	\$4,997,400	\$2,600
2016	\$5,000,000	\$3,345,000	3.00%	\$1,649,900	\$4,994,900	\$5,100
2017	\$5,000,000	\$3,450,000	3.50%	\$1,549,550	\$4,999,550	\$450
2018	\$5,000,000	\$3,570,000	5.00%	\$1,428,800	\$4,998,800	\$1,200
2019	\$5,000,000	\$3,745,000	5.00%	\$1,250,300	\$4,995,300	\$4,700
2020	\$5,000,000	\$3,935,000	5.00%	\$1,063,050	\$4,998,050	\$1,950
2021	\$5,000,000	\$4,130,000	5.00%	\$866,300	\$4,996,300	\$3,700
2022	\$5,000,000	\$4,335,000	4.75%	\$659,800	\$4,994,800	\$5,200
2023	\$5,000,000	\$4,545,000	4.75%	\$453,888	\$4,998,888	\$1,113
2024	\$5,000,000	\$4,760,000	5.00%	\$238,000	\$4,998,000	\$2,000
Total	\$50,000,000	\$39,065,000		\$10,906,988	\$49,971,988	\$28,013

Adjusting for Target Proceeds

39

If there is too much principal (or if there are too many proceeds), reduce target debt service

If there is too little principal (or if there are too few proceeds), increase target debt service

□ To solve for a target par or proceeds amount:

- Make an initial guess for target debt service
- Rescale accordingly

$$DS_{T,1} = \frac{DS_{T,0} \times P_T}{P_0}$$

- ✓ “ $DS_{T,1}$ ” = New target debt service
- ✓ “ $DS_{T,0}$ ” = Initial target debt service
- ✓ “ P_T ” = Target par amount
- ✓ “ P_0 ” = Par amount from initial target debt service
- Iterate, if necessary

$$DS_{T,n} = \frac{DS_{T,n-1} \times P_T}{P_{n-1}}$$

- It may be necessary to adjust by taking the average when within one denomination

$$DS_{T,n} = \left(\frac{DS_{T,n-1}}{P_{n-1}} + \frac{DS_{T,n-2}}{P_{n-2}} \right) \times \frac{P_T}{2}$$

Adjusting for Target Proceeds (Cont'd)

40

Example:

Target principal: \$40,000,000

Coupons: As shown below

Initial target debt service: \$5,000,000

Year	Target Debt				Debt Service	Difference between Target and Actual D/S
	Service	Principal	Coupon	Interest		
2015	\$5,000,000	\$3,250,000	3.00%	\$1,747,400	\$4,997,400	\$2,600
2016	\$5,000,000	\$3,345,000	3.00%	\$1,649,900	\$4,994,900	\$5,100
2017	\$5,000,000	\$3,450,000	3.50%	\$1,549,550	\$4,999,550	\$450
2018	\$5,000,000	\$3,570,000	5.00%	\$1,428,800	\$4,998,800	\$1,200
2019	\$5,000,000	\$3,745,000	5.00%	\$1,250,300	\$4,995,300	\$4,700
2020	\$5,000,000	\$3,935,000	5.00%	\$1,063,050	\$4,998,050	\$1,950
2021	\$5,000,000	\$4,130,000	5.00%	\$866,300	\$4,996,300	\$3,700
2022	\$5,000,000	\$4,335,000	4.75%	\$659,800	\$4,994,800	\$5,200
2023	\$5,000,000	\$4,545,000	4.75%	\$453,888	\$4,998,888	\$1,113
2024	\$5,000,000	\$4,760,000	5.00%	\$238,000	\$4,998,000	\$2,000
Total	\$50,000,000	\$39,065,000		\$10,906,988	\$49,971,988	\$28,013

$$DS_{T,1} = \frac{\$5,000,000 \times \$40,000,000}{\$39,065,000}$$

$$DS_{T,1} = \$5,119,672.34$$

Adjusting for Target Proceeds (Cont'd)

41

Example (cont'd):

Target principal: \$40,000,000

Coupons: As shown below

Initial target debt service: \$5,000,000

Second target debt service: \$5,119,672.34

Year	Target Debt Service	Principal	Coupon	Interest	Debt Service	Difference between Target and Actual D/S
2015	\$5,119,672	\$3,330,000	3.00%	\$1,789,375	\$5,119,375	\$297
2016	\$5,119,672	\$3,430,000	3.00%	\$1,689,475	\$5,119,475	\$197
2017	\$5,119,672	\$3,530,000	3.50%	\$1,586,575	\$5,116,575	\$3,097
2018	\$5,119,672	\$3,655,000	5.00%	\$1,463,025	\$5,118,025	\$1,647
2019	\$5,119,672	\$3,835,000	5.00%	\$1,280,275	\$5,115,275	\$4,397
2020	\$5,119,672	\$4,030,000	5.00%	\$1,088,525	\$5,118,525	\$1,147
2021	\$5,119,672	\$4,230,000	5.00%	\$887,025	\$5,117,025	\$2,647
2022	\$5,119,672	\$4,440,000	4.75%	\$675,525	\$5,115,525	\$4,147
2023	\$5,119,672	\$4,650,000	4.75%	\$464,625	\$5,114,625	\$5,047
2024	\$5,119,672	\$4,875,000	5.00%	\$243,750	\$5,118,750	\$922
Total	\$51,196,723	\$40,005,000		\$11,168,175	\$51,173,175	\$23,548

Attempt	Target Debt Service	Resultant Principal	Solution Method
1	\$5,000,000.00	\$39,065,000.00	Rescale
2	5,119,672.34	40,005,000.00	Rescale
3	5,119,032.46	39,995,000.00	Rescale
4	5,119,352.40	40,000,000.00	Average

Bonus: Excel Functions

Bond cash flows literacy

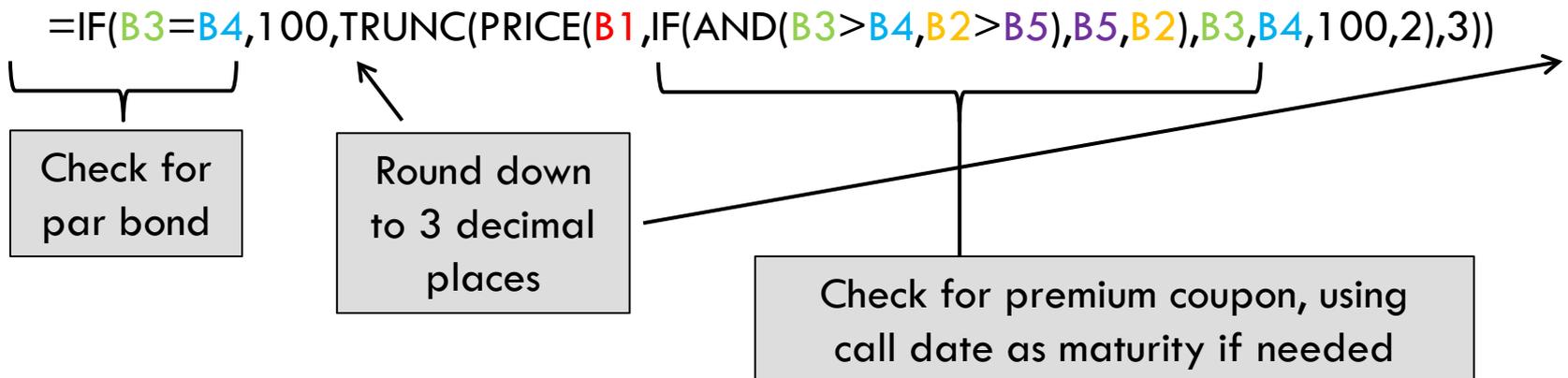
intermediate bond math (Part 1)

Using PRICE()

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- Needs to be supplemented for:
 - ▣ Par bonds;
 - ▣ Rounding; and
 - ▣ Call provisions for premium bonds
- Effective form for bonds callable at par is as follows:

	A	B
1	Delivery	5/14/2014
2	Maturity	5/1/2028
3	Coupon	5.00%
4	Yield	3.65%
5	Call Date 1	5/1/2024
6	Call Price 1	100



Using PRICE()

- For bonds with multiple call prices, must evaluate result for each case

```
=IF(B3=B4,100,TRUNC(MIN(
    PRICE(B1, B2,B3,B4,100,2), PRICE(B1,
    B5,B3,B4,B6,2),
    PRICE(B1, B7,B3,B4,B8,2),
    PRICE(B1, B9,B3,B4,B10,2),
    ),3))
```

	A	B
1	Delivery	5/14/2014
2	Maturity	5/1/2028
3	Coupon	5.00%
4	Yield	3.65%
5	Call Date 1	5/1/2024
6	Call Price 1	102
7	Call Date 2	5/1/2025
8	Call Price 2	101
9	Call Date 3	5/1/2026
10	Call Price 3	100

Using EDATE() and EOMONTH()

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- Used to create regularly aligned dates for principal amortization or debt service schedules

=EOMONTH(A2,5)+1

=EDATE(A6,6)

	A	B	C	D
1	Date	Principal	Coupon	Interest
2	5/14/2014			
3	11/1/2014			\$44,394.17
4	5/1/2015	\$1,000,000	2.00%	47,850.00
5	11/1/2015			37,850.00
6	5/1/2016	1,050,000	3.00%	37,850.00
7	11/1/2016			22,100.00
8	5/1/2017	1,105,000	4.00%	22,100.00

Using SUMPRODUCT()

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- Used to calculate interest for entire bond series (with multiple coupons and principal amounts)

- Tip: Values in last cells must not be blank

	A	B	C	D
1	Date	Principal	Coupon	Interest
2	5/14/2014			
3	11/1/2014			\$44,394.17
4	5/1/2015	\$1,000,000	2.00%	47,850.00
5	11/1/2015			37,850.00
6	5/1/2016	1,050,000	3.00%	37,850.00
7	11/1/2016			22,100.00
8	5/1/2017	1,105,000	4.00%	22,100.00

=SUMPRODUCT(B6:B\$8,C6:C\$8)/2

Using YEARFRAC()

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- Used to calculate interest for irregular periods and for ACT/ACT day count basis

- ▣ Tip: Allows for the same formula to be used the cash flow schedule

	A	B	C	D
1	Date	Principal	Coupon	Interest
2	5/14/2014			
3	11/1/2014			\$44,394.17
4	5/1/2015	\$1,000,000	2.00%	47,850.00
5	11/1/2015			37,850.00
6	5/1/2016	1,050,000	3.00%	37,850.00
7	11/1/2016			22,100.00
8	5/1/2017	1,105,000	4.00%	22,100.00

=SUMPRODUCT(B3:B\$8,C3:C\$8)*YEARFRAC(A2,A3)

Questions?

Thank you for your participation!

A Certificate of Attendance will be emailed to you within a week.

For MCLE credit, please email cdiac_education@treasurer.ca.gov

The video and transcript of this webinar will be available on CDIAC's website in the near future. Please, contact CDIAC if you would like to be notified when they are posted.