California Debt & Investment Advisory Commission

Webinar Transcript
Intermediate Bond Math: Part 1 - Bond Cash Flows Literacy
August 7, 2014

(Editor's Note: This transcript has been prepared by the California Debt and Investment Advisory Commission (CDIAC) and it believes it to be a fair and accurate reproduction of the comments of the speakers. Any errors are those of CDIAC and not the speakers.)

Although municipal bonds are a mainstay of public finance, understanding their economic value requires specialized knowledge and expertise. The vocabulary, financial concepts and calculations, and market incentives may be unfamiliar, if not entirely new, to public agency officials. This two-part intermediate level webinar focuses on the mathematical concepts and calculations underlying bond pricing and structure.

Part 1: Bond Cash Flows Literacy covers the analytics of pricing and builds on the fundamental concepts presented in CDIAC's Debt Essentials seminar, including concepts that form the basis of bond structuring considerations and decisions. This webinar will include understanding yield curves, calculation of debt service, bond pricing formula and pricing conventions, cash flow and amortization schedules, and bond pricing with MS Excel.

Title Slide – Bond Cash Flows Literacy, Intermediate Bond Math (Part 1)

Mark Campbell: Welcome, everyone. This is Mark Campbell, the executive director of CDIAC. I want to welcome you to our webinar, Intermediate Bond Math Part 1. Since this is Part 1, we are expecting to have a Part 2, and I will announce that now. August 20th, same time period, 2:00 to 3:30, we will do Part 2 of this webinar. We've got the presentation slides available on the CDIAC website. Your screen should display the URL for that. So if you are needing those, you may connect through that link. Captioning is provided during the program. Participants may click on the link in the chat section at the bottom of the control panel to access remote captioning. To submit questions throughout the webinar, and I do expect there will be some, please send us your comments or questions in the box marked "questions" near the bottom of the control panel. And, finally, for a certificate of attendance, you must be registered and logged in to the webinar under your own name. A certificate will be e-mailed to all participants at the end of the week. If you require MCLE credits, please e-mail cdiac_education@treasurer.ca.gov.

Polling Question 1 **(01:31)**

I will take a minute to run through some polling questions which will help our speaker, Louis Choi, prepare or modify any of the content here. So let's roll those. I am going to read them off. The first question is: how do you describe your knowledge or experience in regard to the debt financing? Are you a beginner, intermediate, or advanced? We will give you a minute here to respond to that. All right. It looks like we have a fair number of intermediate, and as billed, this is an intermediate course.

Polling Question 2 **(2:39)**

All right. We will go to the next one. Are you directly responsible for debt issuance and administration or oversight in your agency? Take another minute here. Looks like we have a near-even split in responsibilities so at least half of the participants today are directly involved in the debt issuance or administration.

Polling Question 3 **(03:13)**

Let's go to number three here. If you are with a public agency, if you will take a minute to describe your role. Select from the following: debt management and administration, treasury management/investment officer, budget officer, other finance-related, and none of the above. Please select all that apply to you. I think we have the capacity to record that. Ok, so we've got a fair number involved in debt management as well as the treasury management and other finance-related responsibilities.

Polling Question 4 **(04:00)**

And our last polling question. Do you plan to participate in the Intermediate Bond Math Part 2 webinar, which as I announced will be August 20th? Okay. Great. Louis, that sets you up for the expectation that we will have people that will participate in our second program.

Slide 2– Bond Cash Flows Literacy **(04:33)**

Our speaker, Louis Choi, is Senior Managing Director with PRAG. He joined the firm in January 2005 and serves as the primary investment advisor for PRAG's Los Angeles office. He has quantitative analytic experience, including forward-starting swap pricing models, tracking historic performance of municipal variable rate securities versus various swap indices, and structuring refundings with derivative products. Prior to joining PRAG, Mr. Choi was an investment banker in the public finance departments of two national investment banking firms. He has structured over $4 billion in transactions, including both new money and refunding, tax exempt and taxable, fixed and variable rate, and traditional and synthetic structures. So with that, Louis, I am going to turn it over to you and let you take off.

Louis Choi: Thank you very much, Mark. Good afternoon, everyone. Hopefully everybody can hear me fairly well. This is CDIAC's Intermediate Bond Math Part 1 course. It is billed as the Bond Cash Flows Literacy, but we are really going to do a little bit more than just literacy here.

Slides 3 and 4 skipped

Slide 5 – Bonds as Loans **(5:53)**

The topics we will cover today include looking at how bonds are really a proxy for doing loans, how municipal bonds are priced or valued, and understanding the cash flow schedule that people often get, how to do some debt amortization, and then as an added bonus we will cover how to use some Microsoft Excel functions that are related to the above topics. We are going to expect that people will have a variety of questions, and hopefully at the end of each section or after several pages, as the case may be for the longer sections, we will pause and try to answer some of the questions so everybody can try to follow along as best they can. And what you will notice at the end of this presentation is we are not going to cover all topics, all aspects of these topics, as some of the material is quite in-depth, and that is why we have split this training session over two separate webinars.

Without further ado, the first thing we will really talk about is how bonds work in the context as being a proxy of a loan. Bonds serve the dual purpose of being both an investment and a loan, depending on which side the issue one is on. For investors, it is an investment opportunity. And for that, there is a certain time for repayment and rate of return that is commensurate with the risk and opportunity costs for the funds that are invested. And that is what you sort of see graphed here on the top part of the page. That is one aspect of being a bond, a singular bond or group of bonds within a maturity, and that's how it will look like in terms of cash flows for that bond. But it also serves from an issuer perspective, the role of a loan, when bonds are aggregated together where there is a repayment of interest and principal that are structured to be reasonably affordable over time. For most of the issuers perspective who are originating municipal bonds, that is the kind of cash flows and kind of use that they will have, but it is important to understand that it is an investment vehicle on the other side because at the end of the day that is going to help you understand why, or give an appreciation or perspective on why things work the way they do and why the complexities of the math related to bonds exist.

Slide 6 – In Aggregate, Bonds in an Issue Are Equivalent to a Loan (08:37)

These two graphs, obviously, look very different with the investment portion of it being basically a reverse L-shape and the loan portion of it being a square box, but what really can transition between those two things by, as I said, aggregating multiple series of bonds. And then the next slide here we have sort of depicted how one goes about doing that. And here what you will see is that originally starting with the single bond of single maturity that pays both principal in red and interest in gray as you stack on additional bonds -- the top-right graph - and progressively add to that L, over time the aggregate structure will be such that it very much mimics the loan repayment of principal and interest that one is familiar with, with things like mortgages, car loans, and things of that sort, that one sees more commonly on a consumer finance level.

Slide 7 - A Bond Issue and a Loan Are Mathematically Similar, But Not Identical (09:43)

And mathematically, the loan and bond issue aspect reflects all of that as well. One of the things that bonds being an aggregation of multiple different separate securities really expands upon is the ability to have multiple interest rates. And we are trying to show, we tried to show some of the math here that will lead into the rest of the presentation as we take a very much of a deep look at the cash flows and things that go into structuring for bond financings. And then the top part of it, I think, is fairly familiar for folks in the sense that you have a loan, you have a principal balance that gets paid over time and distributed across multiple periods, and along the way you are also paying interest on that loan. The amount of interest over time goes down as the outstanding principal balance declines, but overall, most of the time people structure their debt service payments to be roughly level over time. That is what you see in a typical loan. There is a single interest rate that reads off of all, that is applicable to all the different principal amounts. Whereas for bond issue, one of the complexities is that there are multiple interest rates involved and that each bond, while it is outstanding, will bear interest at that respective rate and that the total interest amount that an issuer pays is the summation of all those individual pieces of interest added together, and the total amount they pay in terms of debt service on the far right-hand side is really a summation of the interest amount as well as the principal that is being amortized.

Slide 8 – Difference in Rates across Maturities Generate a Yield Curve (11:38)

And at that, mathematically the bond issue and loan are similar but they are not identical. And because of that key feature of the bonds really being individual pieces that can have distinct features. And one of the reasons this complexity really exists is that one is able to have different rates over time. And from an issuer's perspective, that's advantageous. In a commercial loan, whether it is a home loan, a car loan, anything of that sort, the interest rates are the same, whether it is the first month's payment or the very last month's payment 30 years from now. You pay the same interest rate on the outstanding principal amount. But in terms of investments and bonds as investments, because of the different risks that tend to expand over time as well as how one measures opportunity costs over time, investments that mature earlier tend to have a lower rate of return than long-term investments. And that's indeed what you see in real life.

And this is one of those examples of this happening. I pulled this from a state public works board transaction that was done back in April of this year. Here, sure enough, you see that the yields at the short end for bonds that were due very quickly were very low. They were less than a quarter percent in comparison to the longest maturities, which were more than 3.5%, more than 4.0%. And, from an issuer's perspective then, it is advantageous to take full advantage of this yield curve, what they call a "yield curve," by structuring the different bond components to have different rates of return and segregating them such that the folks that you are paying back earlier, you are going to offer a lower rate of return than the folks that you are going to pay back a long time from now. And that really from an issuers perspective being able to pay a lower rate even though, for at least some portion of the loan, even though the aggregate borrowing will have a very long term is the chief reason they are willing to put up with a lot of added math that is necessary to model and structure a bond issue. Okay.

And one will notice that the yield curve tends to be most different at the front part of the yield curve. And that also makes sense in the sense that on the short end, when one thinks about it, a one-year investment would have its duration be doubled when buying a two-year bond or tenfold if you are buying a ten-year bond, as opposed to the long end of the yield curve where things tend to be that if you are not getting paid the difference between 24 and 25 years, it is still just a very long time to most folks in people's perception of time, and that is why those yields tend to be very similar at the end side of the yield curve.

Slide 9 – Selected Historical Yield Curves (14:55)

Yield curve is one of those things that is very often talked about in terms of its shape, but it is generally not very firmly defined in terms of where the cutoff marks of what the various shapes are. What I have attempted to do, just to help provide people a guide with this, is to really look at actual historic data and provide some context and some levels that are relative to the near-term history of things. What we are looking at here is a graph of some of an industry benchmark that is provided by Thompson Municipal Market Monitor, or TM3. This is the AAA GO MMD index and it is published on a daily basis, well, each trading day, for each of the 30 maturities that typically exist in many municipal issuances. It reflects the prevailing interest rates of AAA-rated state-level GO bonds. And here we are graphing observations from five different points in time to really show what some of the different yield curves have looked like. Before particular dates, are those dates which have exhibited to some extent the most of that particular behavior, and as well as we have included said rates in orange that reflect rates today.

So four typical shapes that people talk about for yield curves are what people consider normal, which is a slightly ascending line that you see in green; what they also consider steep, which seemingly has been the case for much of the last ten years or more, and that's marked with the blue line; we have also illustrated examples of something being very flat, that's the red line; and slightly inverted, which is in that burgundy line. Now recognize that burgundy – or maybe it is purple – purple line, I recognize that purple line isn't very inverted. It just happens that this historical series of data for AAA GOs have never been very inverted, as opposed to some other yield curves that have existed for treasuries during a particular point in time, especially in the late 2006s, early 2007s, but that is partly a function of the types of risks and opportunity costs and other things that exist in the municipal market, where there is always a natural bias to have an ascending yield curve of some sort. And oftentimes this normal yield curve is already a slightly ascending structure and because of the factors that we just talked about, if for municipal rates it tends to be more ascending than some other yield curves such as treasuries or LIBOR or agencies or corporate bonds or something of that sort. And so sometimes folks talk about there being a steep curve, but there really isn't a great sense as to what is considered steep and away from just being normal, what is considered flat versus what is normal.

So what we attempted to do just to give people a bit of historical context is to look at some history of this data series and to look at not only for those four instances, what the rates and numbers were for comparing two points within the yield curve. So in this first row here, for instance, it is really comparing a one-year maturity versus a five-year maturity, and the second row is one versus thirty, and then the last row here is one versus thirty. And then also show what has been on average the median, which is the fiftieth percentile, and what it has been with the quarter percentile and three quarters percentile. And, arguably, if somebody were to have a yield curve where, for instance, one to five-year is less than 52 basis points, or 0.52 % here, one is in a relative flat area compared to recent history. All the percentiles we have compiled are based on 24 years of data when MMD has been widely available.

Slide 10 skipped

Slide 11 – Time-Value of Money (TVM) (19:55)

This provides some historical context as to what yield curves are. And once again, it is because the existence of these yield curves being able to borrow at different rates, we really look at and structure bonds in sort of this complex math models, take best advantage of that from an issue's perspective and still get a nice flat repayment pattern that one often sees being structured for financings. This first part is just an introduction to why bonds use loans and the particular math. I will pause here for a couple minutes, couple seconds just to see if there are any questions about any of this before we move on to some of the nuanced math related to bonds and cash flow schedules.

Mark Campbell: Louis, at this point we don't have any questions for you.

Louis Choi: All right. Great. The next thing we are going to talk about is how we go about calculating bond prices. This is one of those things fundamental to understanding bond math -- and it is particularly relevant in the sense that municipal bonds tend to make much more use of the different features related to different maturities, different coupons and yields, call features, and things like that, that can affect this math and affect bond prices more so than on other markets. And at the end of the day, that bond price is absolutely one of the key things that drive -- that help factor into what the borrowing cost for an issuer would be.

Now, in terms of calculating bond prices, the basis of all of that is really the time-value of money. So we are going to take a step back and just talk briefly about time-value of money – what it is, how it is represented mathematically, what its general purpose is – before we look into how bonds are being priced.

The time-value of money is really an attempt to value dollars that are in future in the context of what it would be worth today. It is frequently used to measure things like opportunity cost, inflation, investments – not only from a securities perspective but investments in business like buying a new system today to have cost savings over time and things of that sort.

And the mathematical formula for that time-value of money is presented on this slide with the PV, the present value, equal to the future value divided by one plus some kind of interest rate, divided by a compounding period of time or number of periods. The formula is structured this way to take into account the opportunity for people to reinvest interest dividends over time. And it is intended, as the equal sign indicates, to develop equivalents for two dollar values that occur in different points in time.

Slide 12 – TVM IS the Basis for Calculating Bond Prices (23:22)

And in the context of application for municipal bonds, the time-value of money formula can be expanded to take into account multiple cash flows, a whole stream of future cash flows, if you will, by just summing the different pieces of that cash flow together to come up with a single -- summing the pieces of cash flows, each of which happen at different times, to come up with a present value today of the overall stream. And that stream of cash flows is very similar to how bonds repay themselves in the sense that there is periodic repayment of interest, and then at the very end of it all, there is typically the repayment of any remaining interest as well as the final repayment of principal. And all of this leads to multiple terms and for the municipal bonds in particular, it is semiannual interest. And the discounting conventions as well as the time period conventions are adjusted to be a semiannual discounting, that is, two periods every year, and the day count basis is generally 30/360, where in a single year there are exactly 360 days and each month has exactly 30 days. That is just a convention that exists for simplicity for calculations but has held even though nowadays peoples systems can handle much more complicated math.

Slide 13 – Bond Pricing Formula (25:17)

This time-value of money formula is formally translated in the MSRB website in terms of its bond pricing formula where there are components of it to take into account the principal part as well as the interest component but also an accrued interest portion. And this is the official formula for how the MSRB's – the Municipal Securities Rulemaking Board, a self-regulatory body defining the rules related to municipal bonds – sets a description of how prices should be expressed as a function of yields. And all of these prices are expressed as a percentage of the principal amount to be exchanged. The pricing is important because ultimately from an issuing perspective, or I guess an investor's perspective, then this is the exchange of present-day dollars in order to receive future cash flows for holding the bonds or paying interest in principal back on a bond over time.

Slide 14 – Prices Can Vary Greatly with Different Coupons and Maturities (26:28)

The same concept introduced here also can make some of this math a little complex because as I said earlier, there is such a use of different coupons as well as yields and maturities for municipal bonds. On this slide here, we are applying the formula that I have just shown you on the previous page to three different examples. And here in these – and the prices can vary greatly depending on the coupons and maturities that we are talking about. And these three examples will sort of illustrate that.

In each of these instances, we have a ten-year bond – and that's what we are talking about – but in the first example at the top part of the page, we have a bond with a 3.0% coupon and a yield of 3.15%. In that settlement, a bond of ten years with such a coupon and such a yield when calculated out would have a price equal to 98.721% of the principal amount that is going to be exchanged. A similar bond, but not identical bond – one with a higher coupon but a different yield – would calculate out at to a substantially different price. Here you note that the price here to exchange and purchase for that bond with a 5.0% coupon is now 115.764% of the principal amount to be exchanged. And in order for the two prices to be substantially similar, a 5.0% coupon bond would have to have a very different yield than what the 3.0% coupon had. And when you solve it out, you will see that it will have to be 5.165%.

There is a lot of – and all this math is not very intuitive. I mean, there are a lot of digits here. The numbers are not nice and round. And it is for that reason that much of the time pricing isn't discussed necessarily in terms of prices but as yields themselves. In general, people are better able to handle what those yields mean. And we want to talk about a bit all that in further detail a little bit.

And before I leave this page I want to point out one thing because some of these things I will make use of the terminology, and I don't want it to be lost on anyone what it is. A bond in which the price is a hundred is referred to as a par bond. A bond at which the price is less than a hundred, that's a discounts bond in the sense that the face amount that you are going to get back at the end is a hundred percent of the principal amount. And when you are purchasing at a price less than a hundred percent, you are getting it as a discount, if you will. Even though it really isn't because you bought it at a certain yield and all that, but the terminology of the art of municipal bonds uses that kind of wording. And if you were to – if one were to have a bond in which the price is more than a hundred percent of the principal amount, that is referred to as a premium bond.

Slide 15 – Bond Prices are Commonly Expressed in Yields for Ease of Comparison (30:03)

Now, this formula is already very complex looking, but actually there are even more rules related to the pricing of bonds and how that works. And that's what we are going to talk about. Actually, not yet. We are going to talk about first why it is that we use yields in further detail, not only were the prices on the previous pages difficult to handle in terms of the number of digits that are involved, but it also a lot of times it doesn't convey a good sense as to what the appropriate value for an investment is.

And so we saw on an earlier page, there was a general expectation as one invests for a longer period of time for the same credit all being equal, there not being any special rules or things of that sort, then one will get a progressively greater rate of return, and that is what one would expect. And yields help to inform the consistency of pricing across different maturities.

On this page here I've tried to illustrate for folks why that is by showing that if we were just to talk about prices, it is not really something that one can readily grasp whether or not that consistency exists. On this page we have graphed the AAA GO MMD from July 25th of this year, so a couple of weeks ago, and that is represented in the gold line there, and the numbers shown on the bottom there for the first ten years. And as bonds have different prices at different yields, it leads to different prices. And if one were to have – looking at this left-hand example in red here – a two-year bond with a 5.0% coupon and they were told that fair price of that is 109.5%, and they were also informed that a four-year bond with a 3.0% coupon a fair price was 108.5%. A quick look by somebody not appreciating the calculation to yield and all that might lead one to believe that a fair price would be an interpolation between those two prices for a three-year bond with a 4.0% coupon. Because after all, three years is perfectly the middle of two years and four years, and 4.0% is perfectly the middle of 5.0% and 3.0%.

But actually that price of 109 for a three-year bond generates a yield that is substantially – when you graph and look at it on a yield basis – substantially different than what one would expect to be a fair rate of return for a particular bond. And this depiction of how these numbers work out is really why people talk about things in terms of yields because with it one is able to get a true sense of the economics of a particular bond as investment, whereas the prices can sometimes deceive you as maturity ranges change and as coupons change.

The right-hand side shows much the same issue, and here we just have slightly different numbers and slightly different maturities, and it goes to show the same problem in that interpolating things, which is how one would in theory think about it, that doesn't necessarily lead to correct results. And that's why everything is expressed in terms of yields so that across different coupons and nearby maturities, one is able to have a sense as to whether something is a relative fair value.

Slide 16 – Bond Pricing Conventions (34:10)

Now, as I was talking about, the bond pricing formula is already quite complex in having a lot of numbers, inputs into all that, but actually that is not the end story for how municipal bonds are priced. In order for one to speak in terms of yields that are rounded to a fair number of digits after the decimal point – but at the same time to make sure that the amount that ultimately gets exchanged at the end of the day after the agreed-upon yield or pricing – it is important to make sure that all the very specific details like rounding and things of that sort tie out precisely correct. And there are also some nuances in terms of how the formula works when coupon equals yield, as well as the callability of bonds that require additional rules to be put into place.

And there are generally four rules that go into making the adjustments for pricing municipal bonds. The first of those is that whenever the coupon equals the yield, the price is set to be hundred percent. That is just the way it is. If you actually plugged in different numbers into the formula we talked about, sometimes you will get perfectly a hundred but oftentimes you will come up with a solution that is slightly off from a hundred, most often 99.998 or 99.999, which is very close to a hundred, but at the end of a day when you are talking in exchanges of a hundred million dollars, worth multiplying it out, that is $1,000 or $2,000 – $1,000 or $2,000 that they will be off by. That is something that is not desirable so by definition that is true.

And the other aspect of that is when one plugs in numbers to those formulas, they are going to come up with results that look very much like the two examples here where there are many, many digits after the decimal points. And the convention for municipal bonds is truncated to the third place after the decimal so that even though there are all of these additional digits behind here and even though it may be that the third place after the decimal in the second example, the one I am pointing out right now, would normally cause to it round up, by convention, we just cut it off. That's it. We only look at the first three digits and that is all we look at. That is just a convention, and I suspect that probably has to do with the efficiency of early-day calculations for bond prices with certain yields and things of that sort.

Then on the flip side of that, when we are talking about yields, the convention is just to round it to the third place after the decimal. So the fourth digit after the decimal point is more than five, five or more, then the digits would be rounded up, and that is what we show in the example here, in the second example here. In the first example, the number is less than five, so it is rounded down per normal rounding. And it is slightly different in the way prices are treated. That is the third rule.

And the fourth rule is something that happens for municipal bonds because it is so often a feature of municipal bonds to have them be redeemable at a certain price prior to the maturity at the sole discretion of the issuer. And because it is at the sole discretion of the issuer that this happens, certainly for a primary market issuance, it is not unreasonable that the initial investors get the benefit of the doubt in terms of the price, how price is determined, as to guarantee the yields that have been negotiated. And what that really means is that for optionally callable premium bonds – optionally, not mandatory – mandatory too if they also happen to be optional, but certainly the optional is the most important part of it – and they are premium bonds – that is, coupons are greater than yields, something we talked about a couple pages ago – that the bonds are priced to that call date and call price, which is the lowest price. By making sure that the initial investor pays the lowest price, that guarantees that he has the best yield.

Price and yield – though I didn't show it anywhere here other than intuitively displayed with the formula – have an inverse relationship. When you pay a low price for something and you are getting a fixed set of cash flows, the lower your price, the higher your rate of return. And so by doing this such that we, for optionally callable premium bonds such that I have the lowest price, we guarantee that the investors have the best yield. Because once again, it goes back that the investors didn't have any say in whether or not the bonds are optionally redeemed. That is really the sole right of the issuer and as a result of that, the convention is to give the investors the benefit of the doubt. And to make secondary market trades exactly analogous to primary market trades – that is, trades amongst investors – be the same as the first trade between the issuers and investors, the same rule applies. The lowest prices are always guaranteed to the buyer, for the advantage of the buyer, not the seller.

Here, we have illustrated an example of a little bit of a more complicated bond. So a November 1st, 2028 maturity, 4.2% coupon, 3.15% yield, it's callable in about ten years from now – I guess we are settling it on November 1st, 2014, callable in exactly ten years at 102% and then in 11 years at 101% and then in 12 years at 100%. And given all these different permutations and things, one has to evaluate what the price is that would be implied for each of these different scenarios.

Return to Slide 13 – Bond Pricing Formula (41:35)

Scrolling back a couple pages, you will notice one thing that I want to point out, what is talked about here, there is this concept of rather than just principal amount, or a hundred percent, is that there is a redemption value that is used as the basis. As you have different redemption values, meaning if you have call premiums involved, these are the numbers that would be put in place for the purposes of this pricing formula.

Return to Slide 16 – Bond Pricing Conventions (42:03)

And when you work out all the different scenarios, so you assume the redemption date is on the first redemption date at 102%, so that's 20 *(audio cuts out)* periods; or if you are assuming that it happens in 11 years, or 22 periods, at 101%; or 12 years at 100%; or at maturity – so the full 28 periods or 14 years – at 100%; you price out each of those scenarios and you take the one that offers the lowest price. And that is the convention for pricing municipal bonds.

One thing to recognize is many of these bonds are continuously callable. They can be callable any day after the first dates that are stated, but really from a math perspective, that is not only an onerous chore to do the calculations, but it is really only the first dates when the redemption prices change that you have to check. As you move out to later dates, like November 2, 2024, or November 3, 2024, you will find that the prices price higher until the step-down in redemption values. And because of the nature of that, in the full knowledge that is going to happen, that is not necessarily going to change, to track every single period, but it is important to track every single unique period in terms of when redemption prices change and to take the lowest of all these. And in this particular instance, I have picked rates and dates and amounts such that happen to be in one of the middle ones. And that in a nutshell is some of the bond pricing conventions that when coupled with that bond pricing formula goes into figuring out, to translate yields of bonds into prices to be exchanged, upon which the exchange of money and securities would take place.

Slide 17 – Capital Appreciation Bonds (CAB) (43:56)

Now, capital appreciation bonds are also based on sort of the same principals, and the pricing for those work similarly. It is still based on the time-value of money. Basically, the major deviation there is really that the denominations are sold in amounts such that the final value to be repaid at maturity is going to be in denominations of $5,000. And as a result of that, folks have several different ways of expressing that concept. Some people choose to say that the face value is always 5,000 and that the prices are – and that's really probably the most common way – and that the prices are based on fractions of that $5,000. And then people trade in multiples of those $5,000. But the formula works very much the same way in that the value at each instance in time, the Accreted Value over time – the AV here – is really based on the yields divided by two to the power of the number of periods, and that is very similar to what we saw for regular current interest bonds.

Slide 18 – Capital Appreciation Bonds (Cont'd) (45:33)

But keep in mind it is possible for people to express capital appreciation bonds at prices that deviate from what I just described in terms of fractions of the $5,000 denominations. People do sometimes use different accretion values than yields. The accretion rates than yields, that complicates the math, but this is the general formula for doing it. And in the next page, we have just simply shown you a calculation for capital appreciation bonds, how it works to generate an accretion table, and here we have an example of something being delivered May 14, 2014, a maturity of May 1st, 2019, 3.5% yield, it is in a maturing denomination of $5,000. And when you work backwards, you will be able to see all the different accretion values that exist using this formula above and what it was exchanged per $5,000 of maturing value at initial settlement.

Slide 19 – Cash Flow Schedules (46:29)

And that is really a summary of how bond pricing formula works and the conventions related to bond pricing in order to translate yields into the prices. And I will pause here to see if there are any questions from anyone about any of this.

Mark Campbell: Well, again Louis, you are doing such a fantastic job that we don't have any questions for you.

Slide 20 – Describing a Bond Issue with Numbers (47:22)

Louis Choi: Okay. All of that very detailed discussion about exactly how bond pricing works all leads to and flows into how people construct cash flow schedules that are used to describe and memorialize a bond issuance. *(Audio cuts out.)*

Mark Campbell: Louis, did we lose you? Sounds like we did. Can we just hang on and we will get Louis back online here and we will keep going? Louis, are you back with us?

Louis Choi: I am still here.

Mark Campbell: Okay. We lost you for a moment, but we got you back. So go ahead.

Louis Choi: Okay. I apologize. Yeah, I will just start from scratch on this page. I have excerpted three schedules that often appear in official statements or prospectuses for a particular bond issue that describe basically the math that characterizes a set of bonds within that issue. The various cash flows issuers often get are used to generate these tables. And when all is said and done, these terms are the ones that the issuers have to live with for the life of these bonds or until they are redeemed and benefit from the proceeds and things of that sort that are generated and are represented in these three schedules.

And now what we are going to do is really talk about and apply some of the bond pricing that we have been talking briefly about in order to have a better appreciation of how the various numbers in these three schedules relate to one another. And hopefully with that understanding one can better appreciate how to read the schedules and also some of the underlying relationships with what drives what. And the three schedules really are what on an individual maturity basis, the principal amounts, coupons or interest rates – as it is often written in an official statement – the yields of each one – so the bond pricing results – how they relate to the debt service schedule, which shows the principal and interest payments to be made over time and how it also relates to sources and uses, the amount of proceeds that are being generated from the sale of these particular amounts of bonds, and how those proceeds are being applied for different purposes.

Slide 21 – Start with a Basic Loan… (50:40)

So what we are going to start with because, once again, the entire bond issue is really just about having a particular loan, a single loan. So what we are going to start with is just a very basic simple loan, $50 million with a 5.0% interest rate to be repaid in five years. And when you have a typical loan like that, we have the original date of when the loan was made to somebody, when they are expecting to receive payment, and sort of what the payments are each and every single one of those periods. It is possible for one to divide each one of those $11.5 million payments into a principal and an interest portion. And it is such that it would be that over time the principal balance goes down to zero and sums up to the original amount borrowed and still preserve this debt service amount.

Slide 22 – …Round by Denominations… (51:40)

We are going to make small tweaks to this to show the analogy of how this would be constructed through a bond issue with all the nuances related to a bond issue. And the very first thing that one has to tweak because the bonds are securities that can be traded in readily interchangeable denominations, typically in multiples of $5,000, it is to round the principal amounts that are involved to the nearest $5,000.

Slide 23 – …Reflect Different Interest Rates (Coupons) for Each Maturity (52:20)

And that is really necessary in order to make sure that there aren't some leftover pieces for folks that make it difficult for people to exchange them. And really that whole function exists to promote the trading of those securities over time and to make it that each and every single bond are identical to one another from a bondholder's perspective.

After that, we will take into account another thing that we had talked about earlier in a different slide, which is the fact that the bonds themselves can have – because each bond is a separate maturity and has a certainty of payment on that maturity date in terms of principal and then interest leading up to that, all the different times before – there is a possibility to have different interest rates. So rather than the single 5.0% rate that we had shown for the loan, here we show that the 2015 piece has a 2.0% rate, the 2016 piece has a 3.0% rate, the 2017 piece is 4.0%, 2018 and 2019 piece – each one has a 5.0% rate. And what that really means as a result of that, we have different interest amounts that get calculated corresponding to the outstanding principal amounts. So here you see that the $285,000 of interest is being paid for the $9.5 million of principal that will be repaid in 2016. And here we are just aggregating the semiannual payments to show them only yearly. So over time each year there is $285,000 of payments related to that, but because you have five separate bonds, the total amount of interest is all of those added together, and that is what the total interest column is. And frequently you will see this math in this particular section being in the shorthand basis not shown and shown in this more familiar form where you just show principal, coupon, interest, and then the total debt service.

Slide 24 – …Adjust Principal of Each Maturity to Achieve Debt Service Pattern… (54:31)

Now, when you look at that previous slide, one of the things you notice is that the debt service payments aren't perfectly level. And it is possible to rejigger the amount of principal amount in each maturity such that one can get an almost-level debt service profile similar to how the loan works. And the reason that it is not perfectly normal, that it is not perfectly flat and exactly the same each and every single year, is because the principal amounts themselves are rounded to the nearest $5,000. And since the interest rates are multiplied against the $5,000, there is limited control to make it precisely the same across each and every single period. But what you will also notice is that the amount of debt service from the high, from the biggest to the smallest amount, doesn't deviate by more than $5,000 either. And again the bottom part of the page is showing the more familiar forms.

Slide 25 – …Introduce Prices, Yields and Proceeds… (55:48)

Now, the next aspect of that is to recognize that because coupons and yields don't have to be the same, and as a result, prices are not necessarily a hundred percent of the par amount involved, that that has to be introduced as part of structuring a bond issue. And here we are taking some of the ideas and formulas from earlier pages where we show that there are different yields here for each maturity corresponding to different coupons and the resulting prices expressed as a percentage of principal amounts, and then proceeds, which is the product between the principal and price in order to get the proceeds. And that the sum of the proceeds for this $50 million of principal amount is the amount of money that gets netted from the sale of these particular bonds.

Slide 26 – …Calculate Purchase Price… (56:51)

And so in this case because the yields are all, in each maturity, are less than the coupons of the bonds, every single one of the prices are more than a hundred percent. So unsurprisingly, the amount of proceeds is more than the amount of principal shown.

Mark Campbell: Hey, Louis, I've got a question for you.

Louis Choi: Sure.

Mark Campbell: For financial reporting purposes, specifically the MD&A *(management discussion and analysis)* and notes on a financial report, what yield or rate should you use for multiple coupons when you have a bond with multiple coupons?

Louis Choi: We are going to talk about that in this. Typically, what people use is something called the "true interest cost." That will actually be – the calculation of which and the basis for why we do all that will be something that we actually show you how that is done. But in the cash flow package that people put together and report at the conclusion of a bond sale, it will typically have that as one of those statistics. But that is what you would use, typically. There is that, and also depending on the schedule sometimes it makes sense to report it on the average coupon of the bond. And that is also something else that we will show being calculated as well.

Mark Campbell: That was it then. Go ahead.

Louis Choi: Of course, when one goes about selling these bonds, there is a whole bunch of consultants that gets hired, including one of which is the investment bank or the underwriter for the bond, and they will need to be compensated for their efforts. So the proceeds of that $52,642,532 which I have shown you, isn't actually the amount that is going to go into the pocket of the issuer at the end of the sale. Instead, what happens is typically there is an amount they call the "takedown," the compensation used to pay commissions for their salesmen, to keep the lights on at the investment banks, and for their profits, which you typically express in a dollars per thousand dollar amount, what they call "per bond" even though municipal bonds – typically each bond is $5,000, but in the rest of the markets, in the rest of corporate markets, treasuries and things of that sort – the typical denomination is in multiples of $1,000. And as a result of that, the wording is "per bond," even though it is per $1,000 as opposed to $5,000. That is paid to underwriters in addition to certain expenses they may have. So what they have then is the takedown the dollar amounts would be in this case for this maturity $1 per bond, $1 per $1,000 issued. $9,415,000 are issued and if you take $1 per $1,000 of that, that's $9,415. That would occur for each and every single maturity. You can have different takedowns for each maturity. And total amount of takedown that is compensated to the underwriters, sum of those amounts, which in this case is $137,240. In addition to that, there is a variety of expenses that may be involved, all of which they expect to be compensated for. And when you net that amount back, that is the amount that becomes the purchase price of the bond. That is, the underwriter will pay this amount to purchase the $50 million of bonds with these coupons. And when they do that, that reflects the yields at which they will resell them to the public as well as their compensation as well as certain expenses they have incurred as a result of the sale.

Slide 27 – …Add in Sources and Uses Components… (1:01:01)

And then not only does the underwriters discount factor into this, a lot of times there is a need to dispose of different funds, or there may be funds on hand that are available ahead of time to affect the cash flows of the financing. In this particular example here, we assume there was a million dollars of funds on hand as part of this financing, that we have to fund a reserve fund, we have to fund some kind of cost of issuance. And net-net, the production amount here in the aggregate is $53 million and so forth. But if we were to try to get $50 million of project deposit, which is the original principal amount we sort of indicated, after setting aside some of these other funds for the reserve fund, cost of issuance, and underwriter's discount, there isn't quite enough, and this is what results in this negative contingency amount that we see here.

Slide 28 – …and Readjust Principal of Each Maturity to Target Proceeds (1:02:17)

And to fix that, what ends up happening frequently is that the principal amount of bonds is adjusted such that if it were that the contingency's negative – that is, we didn't have enough money – the principal amount are adjusted for each maturity to go up until it is at such a point where the target amount proceeds is what you want and that the contingency amount is a minimal amount, typically this amount will be between zero and $5,000. And at that point, that is sort of all the different aspects of a bond issue put together in terms of how it goes about getting structured to equate a bond issue to a loan.

Slide 29 – How to Calculate the "Yield" of a Bond Issue (1:03:05)

Of course, there is a lot of numbers floating around and all of that. And one of the things that I asked about, just like the question that just came up, was really, well, what was the borrowing cost for the entire loan transaction? That's one of those things. People typically sort of wrap their hands around a total principal amount that was issued is $52 million. The total amount of interest we are going to pay over the entire period is, in this example, $6.9 million. But there is some of that math at the end of the day that people like to see, just like the fact that the prices are not easily understood. Sometimes, it is meaningful for people to see it as a single borrowing cost. And there is a variety of statistics that express that, the three most common of which are what they call the arbitrage yield, the true interest cost, and the all-in true interest cost. And each of those follows the same, generally the same method to calculate, and they are basically the internal rate of return on debt service, or expected debt service on the bonds, to some such that the – that is the internal rate of return which will equate the stream of debt service payments to some target, some initial amount. Typically, that could be – in the case, for example, of true interest cost – what the original exchange between the underwriter, the initial purchaser of the bonds in the issue, would be.

You will recall that the purchase price – that is what that by definition is called – is the total amount of proceeds generated. So principal amount plus the original net premium or discount minus the underwriter's discount. So here, the $54 million that we saw a couple pages back. Such that when you discount the debt service cash flows after the true interest cost here, the 2.65880%, you will see the value equals $54,732,000. There are slightly different rules as to what gets deducted from what to measure people's borrowing cost, but the most commonly used form is the true interest cost here. The arbitrage yield has special rules in it and it is really related for reporting to the IRS in order to preserve the tax-exemption status of many municipal bonds. And the all-in true interest cost people use because people recognize sometimes that not only is the underwriter the only source of deductions from a deposit available to the underwriter, but also cost of issuance is a factor as well. And it follows very much the same scheme, but the target amount that is being used to generate the internal rate of return is slightly different.

Slide 30 – How to Calculate an "Average" (1:06:17)

For averages, which is another related topic, it follows a slightly different method, but it is often a helpful metric used to express things in an aggregate term. And a simpler way of putting it is a weighted average. For instance, when we went to calculate the average coupon, each maturity had a principal amount and a coupon associated with it. To figure out the average coupon, it is simply the sums of the coupon weighted by the principal amount. So each one of these principals would be multiplied by the coupon, as shown by principal multiplied by coupon here in yellow. You add them up and then you divide by the original principal amount to get the average coupon. It is also true for how you figure out what the average weighted maturity is for a typical bond. And those kinds of statistics help summarize basically the terms of an issue. You want to describe this issue, you'd say it was a $52,145,000 issue, the average coupon is 3.85%, the weighted average maturity is 3.069 years, and that in a way encapsulates the whole issue with a single set of statistics.

Slide 31 skipped

Slide 32 – Common Amortization Structures (1:07:45)

Mark Campbell: Louis, I want to give you a time accounting. Just to let you know if we stay on track, we've got about 20 minutes left.

Louis Choi: Thank you. Are there any questions at this time?

Mark Campbell: Not yet. We will keep you posted.

Louis Choi: Okay. One of the things we sort of just glossed quickly over in the previous example where we dived deeply into doing each of the particular numbers is the amortization structure of that particular bond. In that particular bond, we chose one of the most common structures, which is to have level debt service, but there is really a variety of alternatives to doing that. Over the next three slides, we will illustrate six of those different examples. There is no particular magic about any particular one of them. I think the pictures describe exactly what they are, and they are that way for their own purposes.

The first one of these is level principal. And the reason that this was done is really for ease of calculation. It was a very common structure back in the day when people had problems. It was very difficult for people to figure out how to do level debt service on slide rules and things of that sort. And in that structure as a result of that, the same principal amount being paid each and every year and the interest declines over time because the amount of outstanding principal declines. It is still a common method of amortization for bank products with term-out provisions as well as for GOs. And in the former case it is common because the – not that the math couldn't be done nowadays, but it is still easier to do it that way when the amount of draw-down loan, for instance, for a bank product can vary over time. It is just easier to say you are going to make ten equal payments to me in terms principal and the interest. And this is one of those scenarios where because the principal is paid off relatively quickly, the ratio between interest and principal is relatively low.

The second and probably the most common structure nowadays is level debt service. And the reason people do this that it distributes the cost evenly over time and makes long-term budget preparation easy if you have a certain amount of debt service you have to plan for a certain year and you are able to afford it. And then the next year and year after that and the year after that, at least that particular line item in your budget is going to be the same. And in the current market – this is all based on I think a 5.0% rate for 30 years – this will result in an interest rate for a principal ratio that is slightly less than half. And it is really, once again, you pay more principal over time because the amount of interest declines over time. And it is intentionally structured that way.

Slide 33 – Common Amortization Structures (1:11:15)

Another common structure is to defer principal for a small period of time. This is appropriate for those particular financings where maybe something doesn't even get finished for two or three years or whatever it may be, and there may be revenues associated with completing that project. And when that becomes available, then the additional revenues or cost savings could be used and applied to fund the debt service. And that is a commonly appropriate structure for such purposes.

Another scheme that frequently is used is ascending debt service, at least modestly ascending debt service. In this case, we did 2.0% annual growth. And the reason people do this is really that the cost recovery methods and rates you charge, the sales volumes or whatever it might be, the assessed value on the property or something of that sort naturally grows over time or just to reflect inflation. And that becomes a reasonable proxy for doing that. What you will notice is that as the principal amounts are getting pushed later and later, the portion of the payment that is related to the interest in comparison to principal amounts goes up. And that is not surprising. Interest exists as long as principal is not being repaid. And the more -- for the longer period of time which you defer the repayment of principal, the amount of interest grows higher.

And the two other common structures is one that is known as back-loaded principal. In this case we did an extreme, which was to push everything to the very end.

Slide 34 – Common Amortization Structures (1:13:05)

This is commonly done when that particular type of debt ends up being something that people believe has the lowest interest cost for themselves. The idea would be that this front portion here, this empty space part of the L would be filled in with a complementary issue, which would be a borrowing at a higher interest cost as to create a box, if you will, that one would see in a level debt structure anyway. And any opportunities that people believe that they have to arbitrage something with something of that sort, that is a common feature. But once again the interest-to-principal ratio goes up because the principal is being deferred.

And the last of this, similar to the one I just talked about, is wrapping the debt service. And here in this particular example, we have some existing debt service, hypothetically existing debt service for this example, and we structured something so that the whole thing at the end of the day looks like a box. And this is a common strategy for something that – you are refurbishing something so that you extend the useful life of it. And it’s an appropriate application for that. So these are six of the common amortization structures that exist in the marketplace to fit the particular needs of the issuers.

Slide 35 – Solving for Amortization Structure (1:14:24)

The common theme here is that the interest-to-principal ratio goes up over time as things get deferred. And when taking one of these structures, really, there should be a rationale that matches sort of affordability – maybe when revenues come in or something of that sort. That’s good reason rather than, oh, let's defer because we don't want to pay something yet in terms of the principals of good government.

One of the things I had glossed over in going through the detailed math related to the structuring bonds as a loan, which is really how one goes about solving for amortization structure. And on the next few slides, we are going to go just very quickly through how that is done. But essentially it is really goes back to how debt service formula exists where the debt service is equal to the sum of the principal that is being paid, the interest on the principal that is due, as well as the interest on any principal that is still outstanding. And that is this formula here in the top part. And then really figuring out how much principal one would amortize in a given year, provided you know how much debt service you want to pay, it is really rearranging this formula algebraically and then plugging in the various parts you need to solve that equation. And after solving this algebraically and if you have a target debt service number in mind, you know what kind of coupon you need, you’ll be paying for a particular maturity and what other coupons and principal amounts might exist in later maturities, you can go about starting plugging in those numbers.

Slide 36 – Solving for Amortization Structure (1:16:08)

And typically the first part of that number is really at the very end, the very last maturity so you start from the bottom one up, and that's the one place where there are no unknowns. In this particular formula, a lot of terms get reduced to zero in terms of the principal amounts still outstanding after me. The coupons are *(inaudible)* bonds after me. There are none, so it's all zero. And once somebody does that division and you figure out how much principal amount there should be, you do a bit of rounding to the nearest $5,000, which we also talked about, and you can come up if you want to pay $5 million in debt service here and you are paying a 5.0% coupon, that would be having to amortize $4,760,000 in principal that year to get that sum.

Slide 37 – Solving for Amortization Structure (Cont'd) (1:17:01)

And as you go through each earlier maturity, you are provided with all the information you need from the previous solution, and you continue just to solve the question. Here, you will notice that the principal -- the solved principal amount is lower at only $4,545,000, and that reflects that there is $238,000 of interest that must be paid on a later maturity. And the process of solving this is just the exact same as the last step. You are filling in more numbers.

Slide 38 – Solving for Amortization Structure (Cont'd) (1:17:34)

And over time you keep going and repeating the process. And when you do that, you can come up with a principal schedule that fits what the target debt service level is.

Slide 39 – Adjusting for Target Proceeds (1:17:44)

Another aspect of the amortization structure was really that sometimes the target proceeds numbers don't match. And to solve that what one goes about is an iterative process whereby if you end up with too few proceeds, you increase the amount of principal. That makes sense – the more principal you have, the more proceeds you generate. And after you make the necessary deductions for expenses and so forth and other monies to be set aside, it may be that that gets you the right amount. Or conversely, if you have too much proceeds, you will reduce the principal amount you need to issue until that is the case. And what happens is very simply, what one does is, you have the first guess for how much your target debt service amount is and what your target proceeds amount is and what your first result was trying that known debt service number. And to the extent that that doesn't perfectly match when you plug in this formula, that should be your next guess in terms of debt service. And you sort of iterate the process. And sometimes it is helpful if it is too far apart to take steps to interpolate, which is what the bottom part is, to try and help you find a solution. And over time, progressively, one can get to an answer.

Slide 40 – Adjusting for Target Proceeds (Cont'd) (1:19:10)

In this example, we have the same number as we had before. We ended up with plugging in the numbers we needed to have – we wanted to have, I think, $40 million of target principal. It wasn't quite what we want because we only had $39,065,000. So instead of trying the flat $5 million of debt service, which is what we started with, we'll charge slightly more.

Slide 41 – Adjusting for Target Proceeds (Cont'd) (1:19:40)

And as you continue that process, it will get to a point where the resulting principal amount is exactly what you need. And that is how you go about doing that to figure out what the target proceeds amount is.

Slide 42 skipped

Slide 43 – Using PRICE() (1:19:50)

And at this point I can talk about the Excel functions because all of these models work well with Excel, but I am also happy to take any questions that people might have about some of the topics here.

Mark Campbell: Louis, looks like we are still without any questions to feed you. Oh, we are getting something. Hang on. Nope. Sorry, false alarm. Go ahead, Louis.

Louis Choi: All right. All these cash flows here were really built with Excel and I know some of the things here might have seemed a little difficult to follow, especially if this is the first time you see many of these things. But I assure you that with practice and with a little bit of ingenuity and effort, one can certainly figure out how it all works. And I think some of the tools that are necessary to do that, I have attempted to lay out in this series of the most essential Excel functions and how to use them for the particular application that we need.

We will begin with the PRICE function. And really at the end of the day that answers a lot of the problems of what people need to know to figure out how to do prices. And really at the end of the day there is already a built-in price function in Excel, but it is really the additional rules that I laid out in terms of the bond pricing conventions that need to be accounted for that are not built into part of the standardized function. And that is really accounting for par bonds, rounding, and call provisions in the case of premium bonds. And the formula at the bottom here shows all the different terms that are necessary to reflect how to do a price for a standard par bond callable at par and to make sure that it works. So the first part really is that we need to make sure that the coupons and yields are equal. And if they are, the price is a hundred.

We use the TRUNCATE function to round things down to three decimal places when necessary. We do it -- in terms of figuring out what is the correct redemption term to put in, the redemption maturity to put in for the PRICE function, we use the logic of figuring out, okay, 1) you have to be after the call date, and 2) you have to have a premium bond. And if both of those conditions are true, which is what the “AND” does, we will stick in the call date rather than the actual maturity. And with all these fields together in aggregate, this is how you figure out a price for a bond callable at par. And this is a highly flexible formula for doing that.

Slide 44 – Using PRICE ()(Cont’d) (1:23:04)

But if it happens that your redemption terms are more complicated where you have multiple call prices and call dates, it may be necessary for one to individually calculate each and every single price, make a comparison using the minimum functions, doing the truncation, doing the checking for whether or not you have a par bond to have a formula that can accommodate all the provisions necessary to do and figure out price. And that is how all the prices were figured out in the examples that I presented in this presentation. Another aspect –

Mark Campbell: I'm sorry to interrupt. I've got a couple questions if you want to field them now, but we could also wait until the end if you want.

Louis Choi: I can field them now.

Mark Campbell: All right. The first one has to do with term bonds. Apparently, the taxable market prefers these. The question is: why is that the case?

Louis Choi: Term bonds are really structured to provide a trading vehicle for folks. At the end of the day, most bonds that are sold to investors are resold by those investors. Hardly anybody holds something to maturity. There is a lot of trading going on as people face redemptions if they run a bond fund or see market opportunities. And why term bonds exist is to try to create larger amounts. We talked about earlier here where the reason we have all of this complicated math for municipal bonds is because we can take advantage of the yield curve and borrow at lower cost when we intend to repay something earlier. And by and far, the municipal market is the most fragmented market there is in terms of bonds. In the U.S., most corporations except for the very large ones borrow money through banks and syndicated loans and things of that sort. Very few people actually sell securities to have actual bonds. There is only something like 500 corporations that have publicly-traded bonds, U.S. corporations, in comparison to tens of thousands of municipalities that sell bonds. And in order to create a vehicle that is more analogous to what corporate market investors or taxable market investors might have, one has to make the deal seem less fragmented. In a typical -- for instance, in some of these examples I had, a municipal bond issue of 20 years could have 20 maturities if there was only one coupon per maturity. Sometimes, there is more than one coupon per maturity. So a single 20-year loan now becomes 20 pieces. And from an investment perspective, it is difficult for one to find trading partners for exactly the piece you want; however, if it were that all 20 pieces were aggregated to one piece, then you are more likely to find a trading partner because you probably weren't the only guy who bought those particular securities. You can check with the other people that bought those securities to see if they have any interest in buying what you might be holding, and that is why term bonds are such a common thing for taxable bonds. It really has to do with a function of trading. It has to do with the fewness of the credits that exist in the corporate market in comparison to the municipal markets.

Mark Campbell: Okay. So the second question has to do with premium bonds. And the question relates to the difference between the bond proceeds and the principal amount.

Louis Choi: Okay. In a premium bond--

Mark Campbell: So the question really is: why are the proceeds greater than the principal amount?

Return to Slide 12 – TVM Is the Basis for Calculating Bond Prices (1:27:27)

Louis Choi: Okay. When we first talked about how bonds are priced, we talked about how all of it is based on time-value of money. In the municipal bond market, it is quite different than the corporate market in that the coupon on the bond at initial settlement is rarely the same as the yield. In the corporate market, typically when the issue happens, the coupons and yields are the same even though over time as interest rates change the yields can change quickly and things of that sort. But for a variety of reasons, some of them related to counting the tax rules on parts of investors, some of it is just additional hidden economics and things of that sort. The coupons on the bonds for municipal bonds are rarely the same as the yields. And for that matter, for any bond really, as interest rates change, the yield should not be expected to be equal to the coupon. Even for corporate bonds or something of that sort, where initially the coupons and yields are equal, when interest rates change, the yields are going to change over time. The coupons themselves don't change because that is what is memorialized on paper in terms of this is the set of cash flows. This is the basis for calculating how much interest I am going to pay you over time. And it is static. And for fixed rate bonds, it is set and doesn't change over time. So what happens is, for premium bonds you are agreeing to pay somebody a coupon rate that is in excess of the fair, if prevailing, interest rate over time.

Return to Slide 14 – Prices Can Vary Greatly with Different Coupons and Maturities (1:29:27)

So using – let me see. I'm going the wrong way – using this example on this page, the middle example here, the fair price at this point, the fair interest rate for the particular example here is a yield of 3.15%, but you are agreeing that over time you are going to pay them a coupon of 5.0%. Not only are you agreeing to that, but at the end of the day, you are going to pay them back the principal amount that they are saying that they are going to borrow from you. In order for the exchange to be fair, the person would have to give you more than how much you are going to pay them at the end in order for the exchange to be right. Because over that entire period of time the semiannual interest-paying period, you are paying them 5.0% when the fair prevailing market rate is only 3.15%. So that difference is really why in premium bonds – that is, when the coupon exceeds the yield – that the amount exchanged at delivery is expected to be that the buyer pays more than the par, more than 100% of the price of the bonds. Okay?

Mark Campbell: Great. Thanks, Louis. We have one more now having to do with examples – let's see. Please repeat where the slides are located and which examples in your presentation would be good to use the Excel formulas.

Return to Slide 27 – Add in Source and Uses Components (1:31:04)

Louis Choi: All of these starting with the slide 21 here, all of this was built using Excel formulas. Here, I will go through some of the other things, but the dates and how those are computed is all based on one date at the top as opposed to entries. The interest payments and all of those things are calculated using the formulas. The prices here on this page 25 use the PRICE function we just talked about. So all of this example and constructing all of this is very possible through using the Excel formulas that is laid out.

Mark Campbell: Great. That was an excellent way to handle it. Thanks, Louis.

Slide 44 skipped

Slide 45 – Using EDATE() and EOMONTH() (1:31:50)

Louis Choi: Okay. Any other questions?

Mark Campbell: One of your examples included a sinking fund. Could you talk a little bit about what a sinking fund is?

Return to Slide 32 – Common Amortization Structures (1:32:21)

Louis Choi: A sinking fund is that part of the term bond that an issuer obligates itself to repay a certain amount of principal prior to maturity. It is really a structure to allow for sort of the boxing, if you will, as shown in the level of debt service here at the bottom of the page, the boxing, if you will, of a debt service schedule when you have term bonds. If it weren't that you had these sinking funds for a term bond, then one wouldn't have to pay until the very end, and it would be such huge payment. And, in fact, the original municipal bonds or municipal-style bonds for railroads, that was frequently the case. And what ended up happening when the principal was due, those railways went bankrupt, and they didn't get repaid. And to make sure that didn't happen, to make sure there was always sort of a setting aside of money, making sure the investors are being paid over time and to even things out, people decided that yeah, even though I going to have these term bonds that aggregate everything to allow for trading – which answered the first question – we would have these sinking fund provisions where the issuer is forced to repay pieces of it at a time, and that is what a sinking fund is. Any other questions?

Mark Campbell: Not at this time, Louis. And do I think we are butting up against our time limit here. So I will ask you to finish up with what you are able to handle.

Return to Slide 45 – Using EDATE() and EOMONTH() (1:33:48)

Louis Choi: Okay. This is only a couple of slides. And I certainly understand if folks are going to leave us. But if you do have the time, it is helpful, especially if you want to try reconstruct some of the schedules that I alluded to starting on slide 21 before. Here we make generous use of the EDATE and EOMONTH functions. And ultimately, the Excel Help will be very helpful, would be very instructive of how to use all of this by making one aware of the existence of these functions and is also helpful to figuring out how this works. The EOMONTH, the end-of-month function, allows one to put in a date in serial format and then to move to the end of the month, that many months away. So if you had put a zero here, it would be at the end of the same month, but putting a five, it is the end of the month five months away from the original date. And then by adding one you can get the first of the month in the next period. And EDATE here allows for one to quickly, formulaically express a periodicity. And here what the EDATE does is it takes a date in serial date format and offsets it by that many months. So one being one month way, six being six months away to reflect a semiannual interest period. And using these functions, one can create very, very long, long schedules fairly easily without having to rely on Excel to AutoDetect on a drag down or something of that sort and to make the formulas consistent.

Slide 46 – Using SUMPRODUCT() (1:35:25)

Another formula that was used and is helpful as a simplifying step is called SUMPRODUCT. And as the name implies with that, it sums the products of things. So what this is helpful for is in the calculation of interest because once again when one simply expresses interest as the aggregation of the different principal amounts and different coupons that can happen over time, it sometimes gets unwieldy to keep doing the calculation, and SUMPRODUCT helps you by expanding out the number of cells. It gets unwieldy to do that. SUMPRODUCT allows you to judicious use the dollar sign to lock the relative reference of the raise. It lets you quickly be able to structure a calculation for the purpose of calculating interest. And this particular example, the $37,850,000 reflects the principal and coupon, all of it combined. The one tip about this is that this function fails when the last cells are blanks. So keep that in mind if it is not working, that could be the reason why.

Slide 47 – Using YEARFRAC() (1:36:47)

And the last of the formulas that I want to talk about is really YEARFRAC. And this is really helpful for calculating interest and other things where the date is not regular. So if we have a bond that is dated the date of issuance and that doesn't happen to be exactly six months away from the first interest payment, that is often used to sort of figure out how to offset the interest amount by the correct amount to reflect the number of days that weren't part of the period, or maybe were extra, as the case may be. It is also helpful to do accounting for actual/actual basis, and sometimes that's necessary, especially when you're talking about leap years and talking about paying at the beginning of each month and then having to take into account how things work for July, August, things with 31 days versus 30 days versus February and all that kind of stuff where things get irregular. That is an easy way to handle all of that.

Slide 48 – Questions? (1:37:36)

And that is the end of my presentation. I am happy to take any other questions. And if some folks have questions, I am happy to take them if time allows in the next session, at the end of the next session. As well as if you e-mail me, I am happy to answer some questions as well.

Mark Campbell: Louis, we don't have anything at this time. I do want to point out that the Excel formulas and the examples that you provided complement the conceptual discussion with regard to bond pricing, amortization. And I encourage people to utilize those as a way to explore the concepts, not to particularly look to them as tools that you may employ for your work. If there are questions about the spreadsheets themselves, Louis, would you be willing to entertain them from folks?

Louis Choi: Yes, yes. Provided it is not hours and hours, yes.

Mark Campbell: That's what I'm trying to avoid, a search for solutions that people are dealing with in their work specifically. As I said, I think they complement the concepts to be used to better understand the concepts that were presented. Okay. Again, if you desire a certificate of attendance, those will be e-mailed to you, as will a survey, a questionnaire, to help us evaluate the program today. Since we do have a Part 2 coming on, I encourage you to fill out the evaluation and help us resolve any issues that people want to address in the second program. MCLE credits are available again. Email us at cdiac_education@treasurer.ca.gov and we will get those out to you. A video and transcript of the webinar will be available on our website in the near future. We have got to complete the transcription, but the audio version and the transcript will be posted to the CDIAC website under our Education link.

And with that, if we don't have any other follow-on questions – at this point I am getting an indication we don't. So Louis, thank you very much for leading this discussion. And we look forward to Part 2. Again, August 20th at 2:00. You will have to register separately, so if you participated today you will have to go on and register for Part 2.

With that, we will close out the program. Thank you all for your participation. And, Louis, again, thanks for leading the discussion today.

Louis Choi: Thank you.